**Math 140  Lecture 8**

**Definition.** A quadratic function is a degree-2 polynomial $y = ax^2 + bx + c$ with $a \neq 0$.

The graph is a parabola.
- If $a > 0$, the horns point up.
- If $a < 0$, the horns point down.
- If $|a| > 1$, the parabola is narrower than $y = x^2$.
- If $|a| < 1$, the parabola is wider than $y = x^2$.

Find the roots by factoring or using the quadratic formula: 
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$ No roots if $b^2 - 4ac < 0$.

**Completing the Square Theorem.** Every quadratic function may be written in the form: $y = a(x - x_0)^2 + y_0$ where $(x_0, y_0)$ is the vertex (nose) of the parabola.

**Proof.** Factor the $a$ out of the $ax^2 + bx$ part of $ax^2 + bx + c$.

Complete the square. Anything which is added must also be subtracted to preserve equality.

**Examples**
- Find the roots (they are the x-intercepts).
- Write in the form $y = a(x - x_0)^2 + y_0$.
- Graph. On the graph list both coordinates of the vertex.

$y = -\frac{1}{2}(x + 1)^2$

Roots: $x = -1$

$y = -\frac{1}{2}(x - (-1))^2 + 0$

vertex: $(-1, 0)$

$y = 2x^2 - 2x$

$y = 2x^2 - 2x = 2(x - 1)$,

Roots: $0, 1$.

$y = 2(x^2 - x)$

$y = 2(x^2 - x + \frac{1}{4}) - 2 \cdot \frac{1}{4}$

The added $1/4$ is multiplied by 2, thus so is the subtracted $1/4$.

$y = 2(x - \frac{1}{2})^2 + \frac{(-1)}{2}$.

vertex: $(1/2, -1/2)$

$y = x^2 + 2x - 1$

Roots: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{4 + 4}}{2} = -1 \pm \sqrt{2}$

$y = (x^2 + 2x + 1) - 1 - 1$

$y = (x + 1)^2 - 2$

$y = (x - (-1))^2 + (-2)$.

vertex: $(-1, -2)$

**Word problems**
- **Draw the picture. Indicate the variables in the picture.**
- **Write the given equations which relate the variables.**
- **Solve for the wanted quantities. List Given and Answer.**

- The perimeter of a rectangle is 10. Express the area $A$ in terms of the width $x$.

**Picture:**

$$
\begin{array}{c}
\text{Given: } 10 = 2x + 2y \\
A = xy
\end{array}
$$

Want $A$ in $x$, need $y$ in $x$:

$2x + 2y = 10$

$\therefore x + y = 5$

$\therefore y = 5 - x$

$\therefore A = xy = x(5 - x)$

Answer: $A = x(5 - x)$

- The corner of a triangle lies on the line $y = 4 - x$. Express the area $A$ and perimeter $P$ of the triangle in terms of the base $x$.

**Picture:**

$$
\begin{array}{c}
\text{Given: } y = 4 - x \\
A = \frac{xy}{2} \\
P = x + y + z \\
z = x^2 + y^2
\end{array}
$$

Want $A$ in $x$, need $y$ in $x$:

Since $y = 4 - x$, $A = xy/2 = x(4 - x)/2$

Answer: $A = \frac{x(4 - x)}{2}$

Want $P$ in $x$, have $y$ in $x$, need $z$ in $x$:

$z = \sqrt{x^2 + y^2} = \sqrt{x^2 + (4 - x)^2}$

$\therefore P = x + y + z = x + (4 - x) + \sqrt{x^2 + (16 - 8x + x^2)}$

Answer: $P = 4 + \sqrt{2x^2 - 8x + 16}$

- The area of an isosceles triangle is 16. Express the height of the triangle in terms of its width $x$.

**Picture:**

$$
\begin{array}{c}
\text{Given: } \frac{1}{2}xh = 16 \\
A = 16
\end{array}
$$

Want $h$ in $x$.

Answer: $h = \frac{32}{x}$. 