Trigonometric word problems

(a) \textit{Find the exact answer.} (b) \textit{Find the decimal answer.}

To test if your calculator is in radian or degree mode, calculate \( \sin(1) \). \( \sin(1^\circ) = 0.017, \sin(1\text{rad}) = 0.84 \). Give only exact answers on tests.

The bookstore has cheap trig calculators for less than $20.

There are tables for \( \sin \) and \( \cos \) on pages A-34 to A-40.

- Point \( P \) is level with the base of a 1000 ft building which is 4000 ft away. Find the angle of elevation from \( P \) to the top of the building. (a) the exact radian answer, (b) to nearest degree.

\[
\tan(\theta) = \frac{1000}{4000} = \frac{1}{4}.
\]

(a) \( \theta = \tan^{-1}(1/4) \) \( \leftarrow \) exact answer.

(b) \( \theta = 14^\circ \) \( \leftarrow \) to nearest degree.

- An antenna sits atop a 1000 ft building. From point \( P \) on the ground, the angle of elevation to the top of the building is \( \beta \), the angle of elevation to the top of the antenna is \( \alpha \). Express the height of the antenna in terms of \( \alpha \) and \( \beta \).

\[
\tan(\beta) = \frac{1000}{x} \quad \tan(\alpha) = \frac{h + 1000}{x}
\]

\[
x = h \tan(\beta) - 1000
\]

Answer: Height = 1000(\( \frac{\tan(\alpha)}{\tan(\beta)} \) - 1) ft. Remember the units

- From a point \( P \) level with the base of a mountain, the angle of elevation of the mountain is \( \alpha \). From a point \( Q \) 1 mile closer to the mountain’s base, the angle of elevation is \( \beta \). Express the height of the mountain in terms of \( \alpha \) and \( \beta \).

\[
\tan(\beta) = \frac{h}{\sqrt{a^2 - h^2}}
\]

\[
h = a \tan(\beta) - h
\]

Area = \( \frac{1}{2}ab \) = \( \frac{1}{2}(a \sin(\theta))b \) = \( \frac{1}{2}ab \sin(\theta) \) (same answer)

- Find the area of an octagon (stop sign) of radius 1 ft. Give the exact answer and the decimal answer to 2 places.

In each of the 8 triangles, the two sides are radii and have length 1.

\[
\theta = \frac{2\pi}{8} = \frac{\pi}{4}
\]

The area each triangle = \( \frac{1}{2}(1)(1)\sin(\frac{\pi}{4}) = \frac{1}{2}(\sqrt{2}) = \frac{\sqrt{2}}{4} \)

The area of the octagon = 8\( \times \)the area of each triangle = \( 8 \cdot \frac{\sqrt{2}}{4} = 2\sqrt{2} \)

Exact answer = 2\( \sqrt{2} \) sq.ft. Decimal answer = 2.83 sq. ft.