Math 311 Review 2

Exam office hours: Tuesday, 10:30-12:30.
Graded homework is placed in the bin on my office door (PSB 318) and may be picked up after 2:00.
No calculators for this or subsequent exams.
Material Lectures: 9-16. Book: pages 96-151, 156-164.

Definitions

A *vector space, closed* under addition and scalar multiplication, *subspace.* The *null space* and the *nullity* of A.

The *span* of a set of vectors, span S. S *spans* V. Vectors $v_1, v_2, ..., v_n$ are *linearly dependent* or *independent.* A *basis* for a vector space. The *standard basis.* The *dimension* of a vector space V.

$[v]_W$, the *coordinate vector of v w.r.t.* W. $P_{W\leftarrow T}$ the *transition matrix from T to W.*

The *row space (column space)* of a matrix and its *row rank (column rank).*

Theorems

THEOREM. The **0** is unique. So is -u.
LEMMA (CANCELLATION). $u\oplus v = u\oplus v'$ implies $v = v'$.
THEOREM. For any vector space V, any $u\in V$, and scalar c:
(a) $0\circledast u = \mathbf{0}$. (b) $c\circledast\mathbf{0} = \mathbf{0}$.
(c) If $c\circledast u = 0$ then $c = 0$ or $u = \mathbf{0}$. (d) $(-1)\circledast u = -u$
LEMMA(statement only). Given vectors $v_1, v_2, ..., v_n$ written as column vectors of R^m, if $A = [v_1 | v_2 | ... | v_n]$ is the matrix whose columns are $v_1, v_2, ..., v_n$, then $v_1, v_2, ..., v_n$ span R^m iff the rref of *A* does not have a row of 0's.
LEMMA(statement only). Given vectors $v_1, v_2, ..., v_n$ written as column vectors of R^m, if $A = [v_1 | v_2 | ... | v_n]$ is the the matrix whose columns are $v_1, v_2, ..., v_n$, then $v_1, v_2, ..., v_n$ are independent iff the rref of *A* has n nonzero rows.
THEOREM. If $v_1, v_2, ..., v_n$ are a basis for V, then every vector of V can be written in one and only one way as a linear combination of $v_1, v_2, ..., v_n$.
COROLLARY(statement only). Suppose V is a vector space.
(a) If W is a maximal set of independent vectors (i.e., it can't be extended to a larger set of independent vectors) then W is a basis.
(b) If W is a minimal set of vectors which span V (i.e., no proper subset of W spans V) then W is a basis.
THEOREM(statement only). Suppose U and W are subsets of a vector space V and suppose U has u elements and W has w elements.
(a) If U is independent and W spans V, then $u \le w$.
(b) If U and W are both bases, then $u = w$.
COROLLARY. Suppose W is a subset of a vector space V, W has w elements and V has dimension *n*.
(a) W independent implies $w \le n$; $w > n$ implies W is dependent.
(b) W spans V $\Rightarrow w \ge n$; $w < n \Rightarrow$ W doesn't span V.
(c) W independent and $w = n$ implies W is a basis.
(d) W spans V and $w = n$ implies W is a basis.
LEMMA. (a) $P_{W\leftarrow T}[v]_T = [v]_W$ for any vector $v\in V$.
(b) $P_{W\leftarrow T}\cdot P_{T\leftarrow S} = P_{W\leftarrow S}$.
(c) $P_{T\leftarrow W} = (P_{W\leftarrow T})^{-1}$
LEMMA. If $V=R^n$ and U is the standard basis and W and T are any other bases:
(a) $[v]_U = v$ (b) $P_{U\leftarrow W} = (w_1 | w_2 | w_3 ...)$
(c) $P_{W\leftarrow U} = (P_{U\leftarrow W})^{-1}$ (d) $[v]_W = P_{W\leftarrow U}\cdot v$
(e) $P_{W\leftarrow T} = P_{W\leftarrow U}\cdot P_{U\leftarrow T}$
THEOREM(statement only). The rows of a matrix's rref matrix are a basis for the row space. The row rank of a matrix = the number of nonzero rows of its rref matrix.
THEOREM. The row rank equals the column rank of a matrix.

Main Techniques *Be able to*

Identify vector spaces and subspaces.
Determine if a set of vectors span a space, if they are independent, if they are a basis.
Write a vector as a linear combination of other vectors or determine that it is not.
Write parametric equations for a given line.
Find bases for given subspaces or for a null space.
Given a set of vectors: Column method - find a subset which is a basis for the space spanned by the vectors. Row method - find a simple basis for the spanned space.
Be able to find transition matrices and given $[v]_S$ be able to find $[v]_T$ given bases S and T.
Find λ such that $AX = \lambda X$ has a nontrivial solution for a given A.

Suggested Exercises. *All homework exercises plus the recommended excersizes.*

Page Problem
102: 3, 5, 7, 9, 13.
111: 1abcd,3abc,5ab,7abc,9abc,11ab,23abcd,31ab.
122: 1abcd, 3abd, 5, 7, 11abc. 13abc.
137: 1abcd, 3abc, 5bc, 7ab, 9a, 11, 17, 19ac, 21, 29.
147: 3, 5, 13, 15, 17, 19.
161: 1, 3, 7, 9, 13, 19, 23, 25.
169: 1, 5, 19ab, 29ab.

Hw 15 Answers

Page 161

2(2) $\begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$. 4(2) $\begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}$. 8(1) [3,1,3]. 10(1) t^2-3t+2.

14 (a) $\begin{bmatrix} -9 \\ -8 \\ 28 \end{bmatrix}$, (d) $\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$, (b) $\begin{bmatrix} -2 & -5 & -2 \\ -1 & -6 & -2 \\ 1 & 2 & 1 \end{bmatrix}$

(e) $\begin{bmatrix} -2 & 1 & -2 \\ -1 & 0 & -2 \\ 4 & -1 & 7 \end{bmatrix}$, (c) $\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$, (f) $\begin{bmatrix} -9 \\ -8 \\ 28 \end{bmatrix}$.