Math 373 Hw 4 Worked examples and comments.

Hw 147:4.40, 4.44, 4.46, 4.48, 4.50. 155: 4.66. 164: 4.78, 4.80. Rec 147: 4.41-4.51. 155: 4.65, 4.67. 164:4.77, 4.79.

- Suppose a nickel and a dime are tossed.

There are 4 outcomes: {HH, HT, TH, TT} where HT means the nickel was heads and the dime was tails. The first letter represents the nickel; the second is the dime.

The probability of H on the nickel is ½, the probability of T on the dime is also ½, hence the probability of HT is ½×½ = ¼. Likewise HH, TH, TT have probability ¼.

Consider the following events:

A = head on nickel *B* = tail on nickel

C = head on dime *D* = tail on dime

Thus *A*, "head on nickel", is the subset {HH, HT} of outcomes for which the nickel was heads. Likewise *B* = "tail on nickel" = {TH, TT}, *C* = "head on dime" = {HH, TH}.

Clearly P(*A*) = P(*B*) = P(*C*) = P(*D*) = ½.

Using the definition gives the same answer:

P(*A*) = sum of probabilities of outcomes in *A* = P(HH) + P(HT) = ¼ + ¼ = ½.

Mutually exclusive means "disjoint" or "inconsistent" or "can't both happen". *Independent* means "knowing that one event has or has not happened doesn't provide any information about the other.

Are *A* (head on nickel) and *B* (tail on nickel) mutually exclusive? Independent?

They are mutually exclusive, you can't get a head on the nickel and a tail.

They are not independent. If you know that A is true, then you know something about B, i.e., that B is false. In fact mutually exclusive events are never independent.

Are *A* (head on nickel) and *C* (head on dime) mutually exclusive? Independent?

They are not mutually exclusive. HH makes both *A* and *C* true, so *A* and *C* can both happen.

They are independent, know what happened with the nickel does not provide any information about what happened with the dime.

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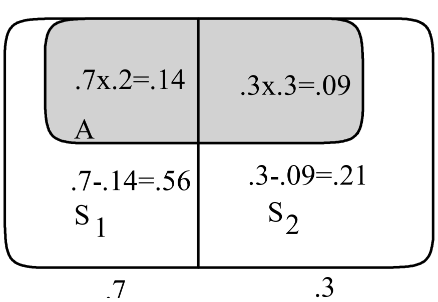
4.65 A sample is selected from one of two populations, S_1 and S_2, with probabilities $P(S_1) = .7$ and $P(S_2) = .3$. If the sample has been selected from s_1, the probability of observing event *A* is $P(A|S_1) = .2$. Similarly, if the sample has been selected from S_2, the probability of observing *A* is $P(A|S_2) = .3$.

First draw the Venn diagram



(a) If a sample is randomly selected form one of the two populations, what is the probability that event *A* occurs?

Just add up the pieces in A.

(.7)(.2) + (.3)(.3) = .23

(b) If the sample is randomly selected and event *A* is observed, what is the probability that the sample was selected from population S_1?

Saying "A is observed" is another was of saying the "A is given". Hence the probability we want is the conditional probability of S_1 given $A = P(S_1|A) = P(S_1 \cap A) / P(A) =$ (.7)(.2) / .23 = .61

(c) If the sample is randomly selected and event A is observed, what is the probability that the sample was selected from population S_2?

This is the conditional probability of S_2 given *A*, $= P(S_2|A) = P(S_2 \cap A)/P(A) =$ (.3)(.3) / .23 = .39

- A red and a green die are rolled. The number that can come up on either die is 1, 2, 3, 4, 5, or 6. You get a "double" if the red and green numbers are the same. If you get a double, you win the total amount on the red and green die. Otherwise you win nothing. What are your expected winnings?

S = {11, 12, 13, 14, 15, 16, 21, 22, 23, 24, ..., 66} where 12 means 1 on the red and 2 on the green. Getting 1 on the red has probability 1/6, getting 2 on the green has probability 1/6, thus getting 12 has probability (1/6)(1/6) = 1/36. Likewise each outcome has probability 1/36.

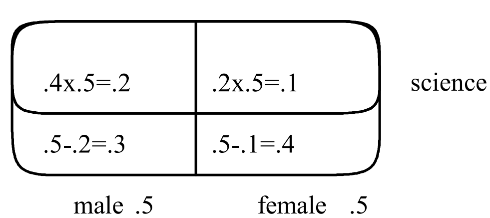
The event of getting a double = {11, 22, 33, 44, 55, 66}. The amounts you win for these outcomes are 1+1 = 2, 2+2 = 4, 6, 8, 10, and 12. For any other outcome such as 12 you get $0.

Expected winning = (amount for 11)(probability of 11)+ (amount for 22)(probability of 22)+ ... + (amount for 66)(probability of 66) + (amount for other outcomes)(probability of other outcomes)

= 2(1/36)+4(1/36)+6(1/36)+8(1/36)+10(1/36)+12(1/36)+0(30/36)

= (2+4+6+8+10+12)(1/36) = 42/36 = $1.67

- .5 of all students are male, .4 of male students are in science, .2 of female students are. What percentage of science students are male?



Percentage = .2/(.2+.1) = .2/.3 = 2/3 = .67 = 67%