Poisson Distributions

In a binomial experiment, there are two outcomes — success/failure, the trials occur a given number \( n \) of times, \( x \) is the number of successes.

In a Poisson experiment the question is not whether an outcome is a success or failure but how often a randomly occurring event happens.

- A fixed period of time or fixed volume of space is given.
- \( x \) is the number of times the event occurs during that period or in that volume,
- \( \mu \), the mean number of times the event occurs, is also given (in binomial experiments we calculate \( \mu \) from \( n \) and \( p \)).
- The event occurrences are random and independent.

The number of lunar eclipses per year is not Poisson (they aren’t random) but the following are: the yearly number of airline accidents, the number of radioactive decays in a gram of uranium per second, the number of times an ATM is used between 6:00 and 7:00pm. With binomial experiments, success/failure occurs in a fixed discrete number of times; with Poisson experiments the number \( x \) of occurrences is discrete but an event can occur anywhere in a continuous period or volume.

Classify the following variables as binomial, Poisson, or neither.

- The number of 911 calls made each day.
- The number of strikes a bowler makes in 10 rolls.
- The yearly rainfall in Manoa.
- Number of drips per hour from a leaky faucet.
- The number of fatalities among the first 100 accidents of the year.
- The number of bacteria in a gram of water.
- The number of chips in a chocolate chip cookie.

An Arizona campsite is observed over a 30-day period each summer. Classify as binomial, Poisson, or neither.

- The number of days the site is burned by a forest fire.
- The number of times the site is burned by a forest fire.
- The number of days the site has campers if campers can stay only one day.
- The number of days the site has campers if campers can stay more than one day.

**Theorem.** If \( x \) is a Poisson random variable measuring the number of occurrences of random independent events in a given period and \( \mu \) is its mean, then

\[
P(x=k) = \frac{\mu^k e^{-\mu}}{k!}
\]

and \( \sigma = \sqrt{\mu} \).

Suppose the average number of chips in a chocolate is 9. Estimate the probability of getting 15 or more chips.

\[
\sum_{k=15}^{\infty} P(x=k) = \sum_{k=15}^{\infty} \frac{\mu^k e^{-\mu}}{k!} = \sum_{k=15}^{\infty} \frac{9^k e^{-9}}{k!}
\]

This is a Poisson experiment with \( \mu = 9 \) and small \( \mu \) = \( np < 7 \); binomial probabilities are approximately Poisson probabilities.

\[
\frac{n^1}{k!(n-k)!} p^k q^{n-k} \approx \frac{\mu^k e^{-\mu}}{k!}
\]

where \( \mu = np \).

We need this fact since Table 1 for binomial probabilities only goes up to \( n = 25 \) and calculators refuse to compute \( n! > 100! \).

1% of a grower’s apples are rotten. In a shipment of 500 apples what is the probability that 0, 1, 2, 3, 4, 5 apples respectively are rotten?

This is a binomial experiment with \( n = 500 \) trials with probability of success (rotten) = .01. Thus the mean \( \mu \) number of expected number of rotten apples is \( \mu = np = 500(.01) = 5 \).

\( n! = 500 \) is too big for either tables or calculators but since the mean 5 is <7, we can regard the number \( x \) of successes as a Poisson random variable with mean \( \mu = 5 \).

<table>
<thead>
<tr>
<th>( k )</th>
<th>( P(x=k) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.007</td>
</tr>
<tr>
<td>1</td>
<td>0.033</td>
</tr>
<tr>
<td>2</td>
<td>0.085</td>
</tr>
<tr>
<td>3</td>
<td>0.14</td>
</tr>
<tr>
<td>4</td>
<td>0.18</td>
</tr>
<tr>
<td>5</td>
<td>0.18</td>
</tr>
</tbody>
</table>