Math 373 Hw 6 Worked examples and comments.

Hw 191: 5.16,5.18 198: 5.30, 5.32, 5.34, 5.38. Rec. 191: 5.17, 5.19. 198: 5.31, 5.35.

- Classify the following variables as Binomial, Poisson, or Neither.

  The number of 911 calls made each day.

  Poisson

  The number of strikes a bowler makes in 10 rolls.

  Binomial

  The yearly rainfall in Manoa.

  Neither, rainfall is continuous the number x of successes (binomial) or the number x of occurrences (Poisson) is discrete.

  Number of drips per hour from a leaky faucet.

  Neither, drips occur rather periodically, not randomly.

  The number accidents each year on H1.

  Poisson

  The number of fatalities among the first 100 accidents of the year.

  Binomial

  The number of bacteria in a gram of water.

  Poisson

  The number of chips in a chocolate chip cookie.

  Poisson provided the cookie batter is not thoroughly mixed.

- An Arizona campsite is observed over a 30-day period each summer. Classify as binomial, Poisson, or neither.

  The number of times the site is burned by a forest fire.

  Neither, burnings are not independent: if it burns today, it will be months before enough tinder has built up to burn again.

  The number of tents put up (assuming no maximum).

  Poisson.

  The number of days the site has campers if campers can stay only one day.

  Binomial

  The number of days the site has campers if campers can stay more than one day.

  Neither. If a camper can stay more than one day, the fact the he is camping today, increases the chances that someone will be camping there tomorrow. Thus the two events of having campers today vs. having campers tomorrow are not independent.

- Suppose the average number of chips in a chocolate is 9. Estimate the probability of getting 15 or more chips

Answer: Suppose the cookie batter isn't thoroughly. Then the distribution is approximately Poisson. An event is the occurrence of a chip, x = number of occurrences = number of chips.

$\mu = 9,\ \sigma = \sqrt{9} = 3.$

Probability of getting 15 or more = $P(x \geq 15) = 1 - P(x \leq 14) = 1 - .959 = .041$.

Suppose the cookie batter is thoroughly mixed but that $\mu = 9$ and $\sigma = 3$. The z-score of 15 is $(15-9)/3 = 6/3 = 2$. The distribution isn't Poisson but still is probably mounded shaped. Hence the best we can do is apply the empirical rule that less than 5% are 2 or more std. dev. from the mean. Henc, in this case, instead of one number, the best we can do is the inequality:

$P(x \geq 15) < .05$.

Suppose it isn't mound shaped, the best we can do is use Chebyshev's rule that at at least $1 - \frac{1}{k^2} = 1 - \frac{1}{2^2} = \frac{3}{4}$ of the distribution are within 2 std. devs. and hence $\leq 1/4 = .25$ is more than 2 std. devs. Hence, in this case, the best we can do is: $P(x \geq 15) \leq .25$

5.19 A home security system is designed to have a 99% reliability rate. Suppose that nine homes equipped with this system experience an attempted burglary. Find the probabilities of the following events:

Answer: This is a binomial experiment. $n = 9$. x = the number of times an alarm was triggered ("success" = triggering an alarm).

(a) At least one of the alarms is triggered = $P(x \geq 1)$

(b) More than seven are triggered. = $P(x > 7)$

(c) Eight or fewer alarms are triggered. = $P(x \leq 8)$

5.31 Let x be a Poisson random variable with mean $\mu = 2.5$. Use Table 2 in Appendix I to calculate these probabilities:

(a) $P(x \geq 5) = 1 - P(x \leq 4)$.

(b) $P(x < 6) = P(x \leq 5)$

(c) $P(x = 2) = P(x \leq 2) - P(x \leq 1)$

(d) $P(1 \leq x \leq 4) = P(x \leq 4) - P(x \leq 1)$