Math 373 Hw 7 Worked examples and comments.

Hw 201: 5.40, 5.42, 5.46. Rec 201: 5.41. 5.43, 5.45.

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5.41 Let x be the number of successes observed in a sample of $n = 5$ items selected from $N = 10$. Suppose that, of the $N = 10$ items, 6 are considered "successes". Find the probability of

(a) no successes= $P(x= 0) = 0$

This is impossible and hence has probability 0.
There are only 4 failures. Hence if $n = 5$ items are selected, at least one must be a success.

Note that $C_1^k = \frac{k!}{1!(k-1)!} = k$. $C_k^k = \frac{k!}{0!k!} = 1$, $0! = 1$.

M=6, N−M=4.

(b) at least two successes = $P(x\geq 2)$ =

$1-P(x=0)-P(x=1) = 1-0-\frac{C_1^M C_{n-1}^{N-M}}{C_n^N} =$

$1-\frac{6C_4^4}{C_5^{10}} = 1 - \frac{6}{\frac{10!}{5!5!}} = 1 - .0238 = .976 \quad = .98$

(c) exactly two successes

$\frac{C_2^6 C_3^4}{C_5^{10}} = \frac{\frac{6!}{2!4!}\frac{4!}{3!1!}}{\frac{10!}{5!5!}} = .238 = .24$

5.45 A company has 5 applicants for 2 positions: 2 women and 3 men. Suppose that the 5 applicants are equally qualified and that no preference is given for choosing either gender. Let x equal the number of women chosen to fill the 2 positions.

Since x counts "success", success is being a woman.
∴ M = number of successes = 2.
N−M = number of failures = 3.
N = the total number of applicants = 5.
The sample = the subset of the the 5 applicants who are choosen for the two positions.
∴ $n = 2$.

(a) Write a formula for p(x), the probability of exactly x successes.

$[C_x^2 C_{2-x}^3]/C_2^5$

(b) What are the mean and variance of this distribution?

$\mu = n\frac{M}{N} = 2\frac{2}{5} = \frac{4}{5} = .8$

$\sigma = \sqrt{n\frac{M}{N}\frac{N-M}{N}\frac{N-n}{N-1}} = \sqrt{2\frac{2}{5}\frac{3}{5}\frac{3}{4}} = \sqrt{\frac{9}{25}} = .6$

From the lecture.

- In a shipment of 100 apples, 5 are rotten. A sample of 4 apples is taken and x is the number of rotten apples in the sample.

x = the number of rotten apples. Thus "success" = being rotten.
N = 100. $n = 4$.
M = number of successes = 5. N−M = 95.

Find the average number of rotten apples in the sample?

$\mu = n\frac{M}{N} = 5\frac{5}{100} = \frac{25}{100} = .25$

Find the std. dev.

$\sigma = \sqrt{n\frac{M}{N}\frac{N-M}{N}\frac{N-n}{N-1}} = \sqrt{4\frac{5}{100}\frac{95}{100}\frac{96}{99}} = .44$

Find the probability that all the sample is rotten?
Note $C_0^k = 1$, there is only one way to pick an empty set.
$C_{k-1}^k = k$, there are k ways to pick a set which omits just one person.

$P(x=4) = \frac{C_4^5 C_0^{95}}{C_4^{100}} = \frac{5}{\frac{100!}{4!96!}} = \frac{5}{4!\cdot 100\cdot 99\cdot 98\cdot 97} = .00$

Find the probability that none are?

$P(x=0) = \frac{C_0^5 C_4^{95}}{C_4^{100}} = \frac{1\frac{95!}{4!91!}}{\frac{100!}{4!96!}} = \frac{95!4!96!}{4!91!100!} =$

$\frac{95\cdot 94\cdot 93\cdot 92}{100\cdot 99\cdot 98\cdot 97} = .81$.

Find $P(x\leq 1)$.

P(x=1)=

$\frac{C_1^5 C_3^{95}}{C_4^{100}} = \frac{5\frac{95!}{3!92!}}{\frac{100!}{4!96!}} = \frac{5\cdot 95!4!96!}{3!92!100!} = \frac{5\cdot 4\cdot 95\cdot 94\cdot 93}{100\cdot 99\cdot 98\cdot 97} = .18$

$P(x \leq 1) = P(x=0) + P(x=1) = .81 + .18 = .99$