Math 373 Hw 8 Worked examples and comments.

Hw 217: 6.2, 6.4, 6.6, 6.10, 6.12, 6.14, 6.16. Rec 217: 6.3ab, 6.5a, 6.11ab, 6.13.

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6.3a′ Find the following probabilities for the standard normal random variable z: 4 decimal places

(a) $P(-1.00<z<.44) = P(-1.00<z<0) + P(0<z<.44) = P(0<z<1.00) + P(0<z<.44) = .3413 + .1700 = .5113$

6.5a′ Find a $z_{.05}$ such that $P(z>z_{.05}) = .025$

If $P(z>z_{.05}) = .05$, then $P(0<z<z_{.05}) = .5 - .05 = .45$

Locate the number nearest .45 inside the normal table, then find value of z. $z_{.05} = 1.69$

If you can't find your number inside the table, use the closest number in the table. We won't bother trying to interpolate between the two closest numbers.

6.10ab′ A normal random variable x has mean $\mu = 20$ and std. dev. $\sigma = 5$. Find the probabilities of these x-values:

(a) $x>23.5$ To get z, subtract the μ and divide by σ.

$$\frac{x-\mu}{\sigma} > \frac{23.5-20}{5} \Rightarrow z > \frac{3.5}{5} \Rightarrow z > .7$$

$P(z>.7) = .5 - P(0<z<.7) = .5 - .2580 = .242 \rightarrow .24$

(b) $x<18.2$

$$\frac{x-\mu}{\sigma} < \frac{18.2-20}{5} \Rightarrow z < \frac{-1.8}{5} \Rightarrow z < -.36$$

$P(z<-.36)$, since the normal curve is symmetric, $= P(z>.36) = .5 - P(0<z<.36) = .5 - .1406 = .3594 \rightarrow .36$

6.14′ A normal random variable x has mean 60 and std. dev. 5. Find a value of x that has area .1 to its right. This is the *90th percentile* of this normal distribution.

$P(z>z_0) = .1 \Rightarrow .5 - P(0<z<z_0) = .1 \Rightarrow$

$P(0<z<z_0) = .5 - .1 = .4$

The number inside the table closest to .4 is .3997. The value of z_0 which gives .3997 is $z_0 = 1.28$

$x_0 = \mu + z_0\sigma = 60 + (1.28)(5) =$ 60.26

Here's a portion of the front-cover table for $P(0<z<a)$.

a	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.00	.00	.01	.01	.02	.02	.02	.03	.03	.04
.1	.04	.04	.05	.05	.06	.06	.06	.07	.07	.08
.2	.08	.08	.09	.09	.09	.10	.10	.11	.11	.11
.3	.12	.12	.13	.13	.13	.14	.14	.14	.15	.15
.4	.16	.16	.16	.17	.17	.17	.18	.18	.18	.19
.5	.19	.20	.20	.20	.21	.21	.21	.22	.22	.22
1.0	.34									
2.0						0.48				

In the problems below, we'll use this two-decimal place table. In you homework, use the four-decimal place table inside the front cover.

- Find

$P(0<z<.3) = .12$

$.3 = .3 + .00$, look up row .3 nd column .00.

$P(-.3<z<.3) = P(-.3<z<0) + P(0<z<.3)$

$P(0<z<.3) + P(0<z<.3) = 2(.12) = .24$

$P(.3<z) = .5 - P(0<z<.3) = .5 - .12 = .38$

Ignore the difference between $<$ and $\le$:

$P(a<z) = P(a\le z)$ since they differ only by a line which has width 0 and hence area 0.

$P(z<.3) = .5 + P(0<z<.3) = .5 + .12 = .62$

Find z_0 s.t. $P(0<z<z_0) = .48$. $z_0 = 2.05$

Find z_0 s.t. $P(z<z_0) = .98$.

$P(z<z_0) = .98 \Leftrightarrow .5 + P(0<z<z_0) = .98$

$\Leftrightarrow P(0<z<z_0) = .48 \Leftrightarrow z_0 = 2.05$

- A student takes a 100 point exam. The average score is 50 with std. dev. 10. Grades are usually a normal distribution.

What is the probability the student gets 55 or more?

We'll do this informally.

$\mu = 50$, $\sigma = 10$

55 is .5 std. devs above normal

Probability of getting 55 or more $= P(z>.5)$

$= .5 - P(0<z<.5) = .5 - .19 = .31$

What is the probability she gets between 45 and 55?

We'll do this one formally.

Probability of getting between 45 and 55 =

$P(45<x<55)$ =, convert from x to z,

$$P\left(\frac{45-50}{10} < \frac{x-\mu}{\sigma} < \frac{55-50}{10}\right) =$$

$P(-.5<z<.5) =$

$P(-.5<z<0) + P(0<z<.5) =$

$P(0<z<.5) + P(0<z<.5) =$, by symmetry,

$2P(0<z<.5) = 2(.19) = .38$

Ans: Probability of getting between 45 and 55 = .38

To get a C, the student must be at or above the 70th percentile. What is the 70th percentile x_0?

$P(z<z_0) = .7$ iff

$P(0<z<z_0) = .2$

Use the table in reverse, find .2 inside the table.

iff $z_0 = .52$

iff $x_0 = \mu + (.52)\sigma = 50 + (.52)10 = 55.2$

Ans. The 70th percentile is 55.2%