Math 373 Hw 12 Worked examples and comments.

Hw 257: 7.30, 7.32, 7.36. Rec 257: 7.29, 7.33abdfg, 7.35abc, 7.37abcd.

Page 257. Recommended exercises

7.27ab Suppose the distribution of potassium in a banana is normally distributed, with mean equal to 630 mg and standard deviation equal to 40 mg per banana. You eat $n = 3$ bananas per day, and T is the total number of milligrams of potassium you receive from them.

For the amount of potassium in each banana,

$\mu = 630$, and $\sigma = 40$

(a) Find the mean and std. dev. of T.

Let x_1, x_2, x_3 be the milligrams of potassium in the first, second and third banana. Thus $T=x_1+x_2+x_3$.

$\mu_T = E(x_1 + x_2 + x_3) = 3\mu = 3(630) = 1890$

$\sigma_T = \sqrt{n}\,\sigma = \sqrt{3}\,40 = 69.2820$ $= 69.28$

(b) Find the probability that your total daily intake of potassium from the three bananas will exceed 2000 mg.

$P(2000 < T) = P(\frac{2000-1890}{69.2820} < \frac{T-\mu}{\sigma})$

$= P(1.5877 < z) = P(z < -1.59) = .0559$

$=.056$

7.29 Random samples of size n are selected from binomial populations with success probabilities p. Find the mean and the standard deviation of the sampling distribution of the sample proportion $\hat{p}$ in each case.

(a) $n = 100$, $p = .3$

$\mu_{\hat{p}} = p = .3$

$\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}} = \sqrt{\frac{(.3)(.7)}{100}} = .04583$ $= .046$

7.35abc One of the ways most Americans relieve stress is to reward themselves with sweets. According to one study, 46% admit to overeating sweet foods when stressed. Suppose that the 46% figure is correct and that a random sample of $n = 100$ Americans is selected.

(a) Does the distribution of $\hat{p}$ have an approximately normal distribution?

Find the mean and mean and std. dev.

$\mu = .46$

$\sigma = \sqrt{\frac{pq}{n}} = \sqrt{\frac{(.46)(.54)}{100}} = .04984$ $= .050$

(b) What is the probability that the sample proportion p exceeds .5.

$P(\hat{p} > .5) = P(\frac{\hat{p}-\mu}{\sigma} > \frac{.5-.46}{.04984}) = P(z > .80)$

$= P(z < -.8) = .2119$ $= .21$

(c) Find the probability that p lies between .76 and .84.

.9513

Comments on Lecture 12

To estimate the proportion p of successes in the population, select a random sample of an appropriate size n and measure the proportion $\hat{p}$ of successes in the sample.

In addition to the original population with probability p of success and the sample population with probability $\hat{p}$ of success, there is the *sampling* population of the proportions $\hat{p}$ of the n-element samples. Different samples have different proportions $\hat{p}$, averaging these proportions $\hat{p}$ over all possible n-element samples gives the mean $\mu_{\hat{p}}$ and std. dev. $\sigma_{\hat{p}}$ of the sampling distribution of $\hat{p}$.

The original binomial population is qualitative rather than quantitative. It has just the two categories “success” or “failure”. Since these aren't numbers, we can't add or average them. What we can do is measure the number of x of successes or the proportion $\hat{p}$ of successes.

The sampling distribution of sample proportions is a quantitative distribution since its measurements are proportions which are numbers. Hence we can take the average $\mu_{\hat{p}}$ and std. dev.

BINOMIAL PROPORTION THEOREM. Suppose n-element samples taken from a binomial population with probabilities p and q of success and failure. Then (a) the mean and std. dev. of the sampling distribution of $\hat{p}$ are:

$\mu_{\hat{p}} = p$, $\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}}$ and

(b) if the expected number of successes and failures in the sample is > 5 (i.e., $np > 5$ and $nq > 5$), the sampling distribution of $\hat{p}$ is approximately normal.

Compare this theorem with the Central Limit Theorem. In the Central Limit Theorem, we are taking averages of quantitative samples. In the Binomial Proportion Theorem, we are taking the proportions of qualitative samples. In the Central Limit Theorem, $n > 30$ guaranteed normalicy. Here we need np and $nq > 5$.

THEOREM. Suppose x and y are independent random variables and a a constant. Then

$E(ax) = aE(x)$,

$\sigma_{ax} = |a|\,\sigma_x$.

COROLLARY. Suppose $x_1, x_2, ..., x_n$ are independent random variables with the same mean μ and std .dev. σ. Then

(a) The sum $x_1+x_2+...+x_n$ has mean $= n\mu$, variation $= n\sigma^2$, and std. dev. $= \sqrt{n}\,\sigma$

(b) (Central Limit Theorem)

The average $\bar{x} = (x_1+x_2+...+x_n)/n$ has mean $\mu_{\bar{x}} = \mu$ and std. dev. $\sigma_{\bar{x}} =$ SE $= \frac{\sigma}{\sqrt{n}}$.

Note the difference between nx_1 and $x_1+x_2+...+x_n$.and $\bar{x}$.

$E(nx_1) = nE(x_1) = n\mu$, std. dev. $= n\sigma$.

$E(x_1+x_2+...+x_n) = n\mu$, but std. dev. $= \sqrt{n}\,\sigma$

$E(x) = \mu$, while std. dev. $= \frac{\sigma}{\sqrt{n}}$

Multiplying x_1 by n increases its deviation from the mean by a factor of n. But adding n independent variables only increases the deviation by a factor of $\sqrt{n}$. This is smaller amount is due to the factor that much of the variations of the independent variables cancel out. When one is increasing, others will be decreasing. If they were not independent and all varied in unison, then the std. dev. would be closer to $n\sigma$ rather than $\sqrt{n}\,\sigma$.