8.21 Find the 90% confidence interval for the population mean \( \mu \).

(a) \( n = 125, \ \bar{x} = .84, \ s^2 = .086 \)

\[ s = \sqrt{s^2} = \sqrt{.086} = .2933 \]
\[ \text{SE} = \frac{s}{\sqrt{n}} \approx \frac{.2933}{\sqrt{125}} = .02623 \]
\[ .84 \pm 1.645(.0262) = [.80, .88] \]

8.23 A random sample of \( n = 300 \) observations from a binomial population produced \( x = 263 \) successes. Find a 90% confidence interval for \( p \).

\[ \hat{p} = \frac{263}{300} = .87667 \]
\[ \text{SE} = \sqrt{\frac{pq}{n}} = \sqrt{\frac{(.87667)(1-.87667)}{300}} = .01898 \]
\[ \hat{p} \pm z_{.05/2}\text{SE} = \]
\[ .8767 \pm 1.645(.01898) = [.85, .91] \]

8.35 Independent random samples were selected from populations 1 and 2. The sample sizes, means, and variances are as follows:

<table>
<thead>
<tr>
<th>pop. 1</th>
<th>pop. 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>sample size</td>
<td>35</td>
</tr>
<tr>
<td>sample mean</td>
<td>12.7</td>
</tr>
<tr>
<td>sample variance</td>
<td>1.38</td>
</tr>
</tbody>
</table>

(a) Find a 95% confidence interval for estimating the difference in the population means \( (\mu_1 - \mu_2) \)

\[ \bar{x}_1 - \bar{x}_2 = 12.7 - 7.4 = 5.3 \]
\[ \text{SE} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \approx \sqrt{\frac{1.38^2}{35} + \frac{4.14^2}{49}} = .3520 \]
\[ (\bar{x}_1 - \bar{x}_2) \pm z_{.05}\text{SE} = \]
\[ 5.3 \pm 1.96(.3520) = [4.61, 5.99] \]

(b) Can you conclude there is a significant difference in the means of the two populations? yes

Why?

Short answer: \( 0 \notin [4.61, 5.99] \) = confidence interval.

Long answer:

The 95% confidence interval \([4.61, 5.99]\) around the difference of sample means \( \bar{x}_1 - \bar{x}_2 \) contains the true difference \( \mu_1 - \mu_2 \) with probability 95%.

\( 0 \notin [4.61, 5.99] \)
\[ \Rightarrow P(\mu_1 - \mu_2 = 0) \leq 5\% \]
\[ \Rightarrow P(\mu_1 - \mu_2) \leq 5\% \]
\[ \Rightarrow \text{equality is unlikely} \]
\[ \Rightarrow \text{the means are significantly different.} \]

8.41 To compare the starting salaries of graduates from education and social science, random samples of 50 recent graduates in each major are selected.

<table>
<thead>
<tr>
<th></th>
<th>mean</th>
<th>std. dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>education</td>
<td>25,554</td>
<td>2,225</td>
</tr>
<tr>
<td>social science</td>
<td>23,348</td>
<td>2,375</td>
</tr>
</tbody>
</table>

\[ \bar{x}_1 = 25,554, \ \bar{x}_2 = 23,348, \ \bar{x}_1 - \bar{x}_2 = 2206 \]
\[ SE = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \approx \sqrt{\frac{2225^2}{50} + \frac{2375^2}{50}} = 460.0445 \]

(a) Estimate the difference in the average salaries of education and social science majors.

\[ \hat{p} = \frac{263}{300} = .87667 \]
\[ \hat{p} \pm z_{.05/2}\text{SE} = \]
\[ 5.3 \pm 1.96(.3520) = [4.61, 5.99] \]

(b) Is there a significant difference between the starting salaries of education majors and social science majors? yes

Why?

Short answer: \( 0 \notin [1304.3128, 3107.6872] \) = confidence interval.

Long answer:

The 95% confidence interval \([1304.3128, 3107.6872]\) around the difference of sample means \( \bar{x}_1 - \bar{x}_2 \) contains the true difference \( \mu_1 - \mu_2 \) with probability 95%.

\( 0 \notin [1304.3128, 3107.6872] \)
\[ \Rightarrow P(\mu_1 - \mu_2 = 0) \leq 5\% \]
\[ \Rightarrow P(\mu_1 - \mu_2) \leq 5\% \]
\[ \Rightarrow \text{equality is unlikely} \]
\[ \Rightarrow \text{the means are significantly different.} \]