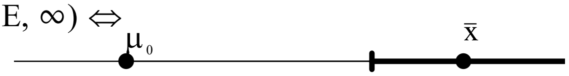
Math 373 Lecture 20

Confidence intervals and acceptance regions

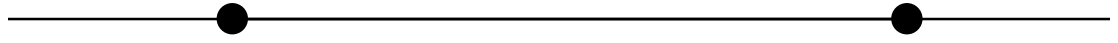
Two-tailed acceptance regions are of the form $\bar{x} \in [\mu_0 - z_{\alpha/2}SE, \mu_0 + z_{\alpha/2}SE]$ and determine which sample means are consistent with the null hypothesis. Confidence intervals are of the form $\mu \in [\bar{x} - z_{\alpha/2}SE, \bar{x} + z_{\alpha/2}SE]$ and determine which population means are consistent with a given sample mean. $\bar{x} \in$ acceptance region $\Leftrightarrow$ $\bar{x} \in [\mu_0 - z_{\alpha/2}SE, \mu_0 + z_{\alpha/2}SE] \Leftrightarrow |\bar{x} - \mu_0| \le z_{\alpha/2}SE \Leftrightarrow$ $\mu_0 \in [\bar{x} - z_{\alpha/2}SE, \bar{x} + z_{\alpha/2}SE] \Leftrightarrow \mu_0 \in$ confidence region.

With two tailed tests, we are concerned about whether or not μ_0 is significantly different from $\bar{x}$. With a one-tailed test we are concerned only about whether one is significantly larger than the other.

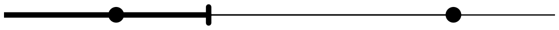
$\mu_0 \notin [\bar{x} - z_\alpha SE, \infty) \Leftrightarrow$



μ_0 is significantly (at level α) less than $\bar{x}$ $\Leftrightarrow$



$\bar{x}$ is significantly (at level α) above μ_0 $\Leftrightarrow$



$\bar{x} \notin (-\infty, \mu_0 + z_\alpha SE]$.

The first interval is the lower bound confidence interval; the second is the acceptance region for H_0: $\mu \le \mu_0$. One-tailed acceptance regions and confidence intervals go in opposite directions.

z-score test statistics

If x is some test statistic with mean μ and std. dev. σ, its z-score is $z = (x - \mu)/\sigma$. One can use either x or z equally well as the test statistic. If x is your statistic, the two-tailed acceptance region is $x \in [\mu - z_{\alpha/2}\sigma, \mu + z_{\alpha/2}\sigma]$. If your statistic is z, then the mean is 0, the std. dev. is 1, and the acceptance region for z is $z \in [-z_{\alpha/2}, z_{\alpha/2}]$. All two-sided tests of significance level α have the same z-acceptance region. When asked to find an acceptance region, assume unless specified otherwise, that it is for x, not z.

■ The null hypothesis is: $\mu = 50$, the alternate hypothesis is $\mu \ne 50$. You have a sample with mean $\bar{x} = 45$ and $SE = 2$. Should you reject the null hypothes at $\alpha = 5\%$?

Since the null hypothesis is assumed to be true, the mean is assumed to be 50. Let z, the z-score of $\bar{x}$, be our statistic. Thus $z = (\bar{x} - \mu)/\sigma = (45 - 50)/2 = -5/2 = -2.5$.

The acceptance region for z is $[-1.96, 1.96]$. Thus $z = -2.5$ is outside the acceptance region and we reject the null hypothesis at the 5% significance level.

■ You wish to show at the 1% significance level that Buicks survive crashes with less damage than Hondas. You crash 40 Buicks and 30 Hondas into a brick wall at 10 mph. An estimate is made of the dollar value of the damage sustained in each crash.

	$\bar{x}_B$	$\bar{x}_H$
Sample size	40	30
Sample mean	\$200	\$210
Sample std. dev.	\$20	\$50

Let μ_B and μ_H be the expected damage for Buicks and Hondas respectively. Note: $\bar{x}_B - \bar{x}_H = -1($

State the null and alternative hypotheses first as an inequality between the means and second, in standard form, as an inequality between the difference and 0.

$H_0: \mu_B \ge \mu_H$ $\quad$ $H_a: \mu_B < \mu_H$

$\mu_B - \mu_H \ge 0$ $\quad$ $\mu_B - \mu_H < 0$

Null Region: $\mu_B - \mu_H \in [0, \infty)$. $\quad$ $SE = \ \dots\ = 9.6609$

The acceptance region for $\bar{x}_B - \bar{x}_H$:

$\bar{x}_B - \bar{x}_H \in [0 - 2.33(9.66), \infty) = [-22.51, \infty)$

The confidence region for $\mu_B - \mu_H$:

$\mu_B - \mu_H \in (-\infty, -10 + 1.645 * 9.66] = (-\infty, 5.89]$

Now let's do the problem a second time, this time using the z-score of the sample differences as your test statistic. What is the value of our test statistic?

$z = ((\mu - \mu_0) - 0)/SE = -1.035$

The null region for z: $[0, \infty)$

The acceptance region for z: $z \in [-1.645, \infty)$.

Accept H_0? Yes, $-1.03 \in [-1.645, \infty)$.

Binomial proportion tests

Suppose you are testing a null hypothesis that a binomial proportion p is some null proportion p_0. You run the experiment n times and get a sample proportion $\hat{p}$.
Hypotheses: H_0: $p = p_0$ and H_a: $p \ne p_0$.
Test statistic: Let's use the z-score of $\hat{p}$.

Since we assume the null hypothesis, we use p_0 for the population mean. Thus the z-score is $(\hat{p} - p_0)/SE$. Likewise for SE, by the assumption of the null hypothesis, the correct formula for SE is $\sqrt{\frac{p_0 q_0}{n}}$ not the sample value $\sqrt{\frac{\hat{p}\hat{q}}{n}}$.

■ A self-proclaimed psychic claims he can predict coin tosses correctly at least 55% of the time. You toss $n = 100$ coins and he correctly predicts 50 of the tosses. Does this refute his claim at the 5% significance level?

Let p = the percentage of time the psychic is correct.

H_0: $p \ge .55$. H_a: $p < .55$.

$SE = \sqrt{(.55)(.45)/100} = 4.97\%$

Use a z-score as your test statistic.

Null region: $z \in [0, \infty)$

The z-score: $\dfrac{\hat{p} - p_0}{SE} = \dfrac{.5 - .55}{4.97} = -1.005$

The acceptance region for z: $z \in [-1.645, \infty)$

Reject H_0? No, $z = -1.005 \in [-1.645, \infty)$.