10.19. Independent random samples of \( n_1 = 16 \) and \( n_2 = 13 \) are selected from two normal populations. We wish to test if the distributions and their means differ. Let the significance level be 1%.

<table>
<thead>
<tr>
<th></th>
<th>Pop 1</th>
<th>Pop 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample size</td>
<td>16</td>
<td>13</td>
</tr>
<tr>
<td>Sample mean</td>
<td>34.6</td>
<td>32.2</td>
</tr>
<tr>
<td>Sample std. dev.</td>
<td>2.2</td>
<td>2.4</td>
</tr>
</tbody>
</table>

(a) State the null and alternate hypotheses.

\[ H_0: \mu_1 = \mu_2, \quad \mu_1 - \mu_2 = 0, \quad H_a: \mu_1 \neq \mu_2, \quad \mu_1 - \mu_2 = 0. \]

(a') Find the pooled std. dev. \( s \) and the standard error.

Note: \( t \) is calculated the same way as \( z \) with the \( z \)-score.

\[
s = \sqrt{\frac{15(2.2)^2 + 12(2.4)^2}{16 + 13 - 2}} = 2.2910
\]

\[
SE = 2.2910 \sqrt{\frac{1}{16} + \frac{1}{13}} = 0.8555
\]

\[ df = 16 + 13 - 2 = 27 \]

(b) Find \( t \) and the \( \alpha=1\% \) acceptance region for \( t \).

\[ t = \frac{(34.6 - 32.2)/0.8555}{2.805} \approx -2.805 \]

acceptance region: \( t \in [-2.771, 2.771] \)

(c') Find the 99% confidence interval for \( \bar{x}_1 - \bar{x}_2 \).

\[ \mu_1 - \mu_2 \in 2.4 \pm 2.771 \times 0.8555 = [0.030, 4.770] \]

10.25. An experiment tests the effectiveness of ethanol and bleach solutions as disinfectants. In each run of the experiment, 5 plant cuttings were cultured for 4 weeks and the number of uncontaminated cuttings were then counted. There were 15 runs with the following results.

<table>
<thead>
<tr>
<th></th>
<th>1. ethanol</th>
<th>2. bleach</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>3.73</td>
<td>4.8</td>
</tr>
<tr>
<td>std. dev.</td>
<td>1.67</td>
<td>0.41</td>
</tr>
<tr>
<td>( n )</td>
<td>15</td>
<td>15</td>
</tr>
</tbody>
</table>

We want to know if there is a significant difference.

\[ H_0: \mu_1 = \mu_2, \quad \mu_1 - \mu_2 = 0, \quad H_a: \mu_1 \neq \mu_2, \quad \mu_1 - \mu_2 = 0. \]

At this point determine if you should or shouldn’t use the standard error with pooled sampled deviations.

\[ SE = \sqrt{\frac{1.67^2}{15} + \frac{0.41^2}{15}} = 0.4440 \]

\[ t = \frac{(3.73 - 4.8)/0.444}{2.4099} \]

(b') Find the number of degrees of freedom \( df \).

\[ v_1 = \frac{1.67^2}{15} = 0.1859, \quad v_2 = \frac{0.41^2}{15} = 0.0112 \]

\[ df = \frac{(v_1 + v_2)^2}{v_1/14 + v_2/14} = 15.68 \to 16 \]

(c') Find the acceptance region for \( t \).

acceptance region = \[ -2.12, 2.12 \]

(d) Is there significant evidence that the solutions differ? \[ \text{yes} \]

(e) Why? \( t = -2.4 \notin [-2.12, 7.12] \) = accept. region for \( t \).