Math 373    Review 3

Exam office hours: Thursday, 10:30-12:30. Exam Fri. Graded homework is placed in the bin on my office door (PSB 318) and may be picked up after 2:00.

Bring scratch paper and your calculator. Know \( z_\alpha \) and \( z_{\alpha/2} \) for \( \alpha = 5\%, 1\% \). 1.645, 2.33, 1.96, 2.58.

Definitions

**Definition.** The *p-value* of an estimator \( e \) is the probability of being as far or farther from the null region as \( e \).

**Definition.** Type I error: Falsely rejecting \( H_0 \).
Type II error: Falsely accepting \( H_0 \).

**Definition.** Let \( \beta = P(\text{falsely accepting } H_0) = P(\text{accepting } H_0 \text{ when it is false}). \)
The *power* of the test is \( P(\text{correctly rejecting } H_0) = P(\text{accepting } H_a \text{ when it is true}). \)

Student's *t*: Let \( t = \frac{\bar{x} - \mu}{s/\sqrt{n}} \). For small samples, \( n < 30 \), Student's *t* differs significantly from the normal distribution of *z*. The shape of the distribution for *t* depends on the number of degrees of freedom. For an *n*-element sample, \( df = n - 1 \). For a two population sample, \( df = n_1 + n_2 - 2 \).

**Definition.** Let \( t_\alpha \) be the point whose right tail \( [t_\alpha, \infty) \) has probability \( \alpha \) under Student's *t* distribution.

Theorems

**Theorem.** The probability of falsely rejecting \( H_0 \) is the level of significance \( \alpha \).

**Theorem.** The power of the test is \( 1 - \beta \).

For tests involving binomial proportions, \( SE \), under the assumption of the null hypothesis, is best estimated with

- \( SE = \sqrt{pq \left( \frac{1}{n_1} + \frac{1}{n_2} \right)} \) where \( p = \frac{x_1 + x_2}{n_1 + n_2} \) is the pooled proportion.

Using pooled vs. unpooled standard error for the difference \( x_1 - x_2 \) of two means: assume \( \sigma_1 = \sigma_2 \) unless the ratio of the larger of the two variances \( s_1^2, s_2^2 \) over the smaller exceeds 3.

If not assuming \( \sigma_1 = \sigma_2 \), use the unpooled

- \( SE = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \) with the following degrees of freedom:
  - \( df = \frac{(v_1 + v_2)^2}{\frac{v_1^2}{n_1 - 1} + \frac{v_2^2}{n_2 - 1}} \) where \( v_1 = \frac{s_1^2}{n_1}, v_2 = \frac{s_2^2}{n_2} \).
  - Round \( df \) to the nearest integer.

If assuming \( \sigma_1 = \sigma_2 \), use the pooled estimate

- \( SE = s \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \).
  - where \( s = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}} \).

**Definition.** \( \chi^2 = \frac{(n-1)s^2}{\sigma^2} \) is the chi-square variable for the sample variance \( s^2 \). The distribution is in Table 5.

Main Techniques Be able to
Do all homework type problems.

Suggested Exercises. *All homework exercises plus the recommended exercises.*

Page Problem
335: 9.1, 9.7, 9.9, 9.11.
373: 10.1, 10.3, 10.5.
383: 10.17, 10.19, 10.25.
390: 10.29, 10.31, 10.33, 10.35.
400: 10.45, 10.49.