Recommended are not to be turned in. Each problem differs in some way from the text problem. Turn in this sheet, not your scratch paper.

Scilab notation. (Scilab is the software package you will be using starting Wednesday.)

\[
\begin{bmatrix} a & b & c; d & e & f \end{bmatrix} = \text{the 2}\times3 \text{ matrix }
\begin{bmatrix} abc \\
\end{bmatrix}
\]

\[A' = A^T = \text{the transpose of } A. \quad A^*B = AB\]

\[\text{eye}(n, n) = I_n = \text{the } n\times n \text{ identity matrix with 1’s on the diagonal and 0’s elsewhere.}\]

\[\begin{bmatrix} X; Y \end{bmatrix} = \begin{bmatrix} X & Y \end{bmatrix} = \begin{bmatrix} X | Y \end{bmatrix} = \text{the matrix consisting of the rows of } X \text{ followed by the rows of } Y.\]

Note. If a matrix for a system of equations has a row which is all zeros except for the constant column, the system has no solution. If the variables are \(x, y\), then \[\begin{bmatrix} 123 \\
001 \\
0=1 \end{bmatrix}\] corresponds to \(x+2y=3, 0=1\). No matter what \(x\) and \(y\) are, \[\begin{bmatrix} 123 \\
001 \\
0=1 \end{bmatrix}\] is never true. No solution exists.

The problems are stated in Scilab form, but write your answers in the usual Math 311 rectangular form.

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\[A = \begin{bmatrix} 2 & 3 & 1; 3 & 2 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 0; 3 & 2; 1 & 2 \end{bmatrix},\]

\[C = \begin{bmatrix} 2 & -1 & 3; 4 & 2 & 6; 3 & 2 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 3 & -1; 2 & -3 \end{bmatrix},\]

\[E = \begin{bmatrix} 2 & 3 & -2; 0 & 2 & 5; 1 & 2 & 3 \end{bmatrix}.\]

Compute the following if possible; write “not possible” otherwise. Numbers should be integers in \([-38, 58]\); 9 negatives.

2(5). (a) \(C+E\) \quad (b) \(AB\) \quad (c) \(A^*(B*D)\) \quad (d) \(A^*(D+E)\)

(c) Write the \(3\times4\) augmented matrix.

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See Lecture 1 for the definition of tableau row operations, pivot operations, tableau normal form, and for the tableau normal form algorithm. Unlike elementary row operations, you may not interchange rows.

2’(3). Transform to tableau form (not rref as in text).

\[A = \begin{bmatrix} 1 & 3 & 2 \ 1 & 2 & 2 \ 1 & 4 & 1 \ 1 & 5 & 2 \end{bmatrix}; \quad B = A.\]

Suggestion: save \(A\) to \(Z\) (\(Z = A\)) for backup and save \(A\) to \(B\) (\(B = A\)).

List the matrix after the 1st 2 steps of the completing pivot on column 1. List the matrix after completing the 2nd column pivot operation. Suggestion: use p.

\[\begin{array}{cccc}
1 & 3 & -1 & 2 \\
0 & ___ & ___ & ___ \\
0 & ___ & ___ & ___ \\
1 & 5 & 3 & 2
\end{array}\]

List the final answer. Should have 3 identity columns.

Suggestion: If you saved \(A\) to \(B\) as above, calculate rref(\(B\)), it should have the same rows, in some order, as your answer.

4(3). Use pivot operations to transform the following to tableau normal form. Show the intermediate matrix formed after completing the 1st column pivot. Then give the final rref answer. (You should even pivot on the "constants" column if needed.)

Use decimals 1.75 instead of fractions 7/4 or 1 ¾.

4(3).

\[\begin{array}{cccc}
1 & 0 & 2 & 3 \\
4 & -4 & 1 & 1 \\
-2 & 4 & 3 & 5
\end{array}\]

1st: one 0 \quad 2nd: three 0's \quad 4th: eight 0's

(a) Write the \(3\times3\) coefficient matrix.

(b) Write the linear system as a matrix equation \(C\cdot X = B\).
See Lecture 1 for the definition of basic and general solution.

6(6). Find the basic solution and the general solution. Write “no solution” if there is none.

(a) \[ x + y + 2z = -3 \]
\[ 2x + 2y - 5z = 15 \]
\[ 3x + y - z = 10 \]
\[ 2x + y + 2z = 5 \]

Basic sol.:

General sol.:

(b) \[ x + y + 2z + w = 4 \]
\[ 2x - 2y + 3z - 2w = 5 \]
\[ x + 7y + 3z + 5w = 7 \]

Basic sol.:

General sol.:

14(4). \[ x + 2y - 2z = 4 \]
\[ -y + 5z = 2 \]
\[ x + y + (a^2 - 13)z = a + 2 \]
Find all \( a \) (write “no such \( a \)” if none) such that

(a) there is no solution,

(b) there is a unique solution,

(c) there are infinitely many solutions.

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Compute the inverses \( B \) (if any) of the matrix \( A \) by converting the matrix
\[ [A | I] \]
to reduced row echelon form \([I | B]\). List the intermediate matrix \([C,D]\) which follows the completion of the first pivot step. Use two place decimals.

6(4). \( A = \begin{bmatrix} 2 & 5 & 1 & 0 \\ 3 & 2 & 0 & 1 \end{bmatrix} \)

\begin{array}{cccc}
2 & 5 & 1 & 0 \\
3 & 2 & 0 & 1 \\
\end{array}

-5.5
-0.27

New(2). Suppose the matrix \[ [1 \\ 0 \\ 0; 0 \\ 1 \\ 4; 0 \\ 0 \\ 0] \] is the tableau form of the augmented matrix for a system of equations in \( x, y, z \). There are two possibilities for the set of basic variables. List both.

\{ 0, 0 \} \quad \{ 0, 0 \}