

Math 414 Lecture 1

Homework assignments are always due at the beginning of the next lecture.

“Operations Research” studies linear programming and associated algorithms. Businesses use it to find resource allocations and production schedules which yield maximum profit. Linear programming solves problems which can be modeled with linear equations and inequalities.

Reading assignment: Text: Pages 1-29.

Install SciLab/Octave/Matlab or OldMatlab read webpage's manuals.

You must learn Scilab notation.

For matrices A, B and real r :

$A+B, A-B, A/r$ are what you would expect.

$A*B = AB, r*A = rA. \det(A) =$ the determinate of A .

$A' = A^T =$ the transpose, $\text{inv}(A) = A^{-1} =$ the inverse matrix,

$\text{eye}(n, n) = I_n =$ the $n \times n$ identity matrix (1's on diagonal, 0 elsewhere),

$[A, B] = [A | B] =$ matrix A followed by matrix B .

$A(i, j) = ij^{\text{th}}$ entry in $A, A(i, :) = i^{\text{th}}$ row, $A(:, j) = j^{\text{th}}$ column.

$[1 \ 2 \ 3]$ is a row vector, $[3; 6; 7]$ a column vector.

$[1 \ 2 \ 3; 4 \ 5 \ 6]$ is the 2×3 matrix $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$

What is displayed after entering these lines?

$A = [1 \ 2 \ 3; 4 \ 5 \ 6; 7 \ 8 \ 9], A(2,3), A(2,:), A([2,1],:).$

What happens to A when you enter the lines below?

$A(1,:) = 2*A(1,:), A(1,:) = A(1,:) - 2*A(2,:), A([1,2],:) = A([2,1],:).$

To pivot on the row 2, column 3 entry of matrix A ,

Load the pivot program 'p', by entering:

`load('414')` This needs to be done only once.

Then enter: $r=2; c=3; \text{execstr}(p), (\text{eval}(p) \text{ or } >p<)$

■ For the system of equations, $r: ax + by = c$
 $s: dx + ey = f$

The coefficient matrix is $[a \ b; d \ e].$

The matrix equation is $[a \ b; d \ e]*[x; y] = [c; f].$

The augmented matrix is $[b \ c; d \ e \ f]$

$[a; d] = x$ column, $[b; e] = y$ column, $[c; f] =$ constant column.

DEFINITION. The tableau row operations are:

- Multiply a row by a nonzero constant,
- Add a multiple of one row to another row.

The elementary row operations consist of the above plus

- Interchange two rows.

In an identity column, one entry is “1”, the rest are “0”.
E.g. $[0; 0; 1; 0]$.

THEOREM. Applying these row operations to (the augmented matrix for) a system of equations does not change the set of solutions for the system.

A matrix is in tableau form if the number of independent identity columns equals the number of nonzero rows. The basic variables are those associated

with the independent columns. All other nonbasic variables are parameters.

A subset of the variables is basic if its set of columns is independent.

A tableau matrix is in reduced row echelon form (rref) if the “1”’s of the basic variables form a decreasing line of leading nonzero entries and any all zero rows are at the bottom.

Associated with every tableau is a basic solution in which the parameters are zero. The general solution writes the basic variables in terms of the parameters which are left unassigned, i.e. the parameters are arbitrary.

■ Suppose the variables are $x, y,$ and z . Then the following augmented matrix is in tableau form. Rearranging the rows gives the rref form on the right. x, y the basic variables and z is nonbasic.

$$\begin{matrix} x & y & z & & \\ \begin{bmatrix} 0 & 1 & 4 & -2 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 3 & 8 \end{bmatrix} & \text{tableau} & \text{rref} & \begin{bmatrix} x & y & z \\ 1 & 0 & 3 & 8 \\ 0 & 1 & 4 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix}$$

rref system of equations: $x + 3z = 8, y + 4z = -2, 0 = 0$.

Basic solution: $z = 0, x = 8, y = -2, z$ is the parameter.

General solution: $x = 8 - 3z, y = -2 - 4z, z$ is arbitrary.

To pivot on a nonzero entry in row r and column c of a matrix A , use the first tableau operation to set the entry to 1, use the second tableau operation to set the other entries in column c to 0. In the example below we pivot on the row 2, column 1 entry. To do this pivot in Scilab, enter:

$r=2; c=1; \text{execstr}(p)$

$$\begin{matrix} \begin{bmatrix} 1 & 3 & 0 \\ (2) & 4 & 1 \\ -2 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 1 & 3 & 0 \\ (1) & 2 & .5 \\ -2 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 0 & 1 & -.5 \\ (1) & 2 & .5 \\ -2 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 0 & 1 & -.5 \\ 1 & 2 & .5 \\ 0 & 4 & 2 \end{bmatrix} \end{matrix}$$

The pivot operation produces an identity column.

TABLEAU NORMAL FORM ALGORITHM. To reduce a matrix to tableau form:

Pivot on the 1st nonzero entry in the first nonzero row.

Pivot on the 1st nonzero entry in the next nonzero row.

...

■ In the example below the pivot points are in ().

$$\begin{matrix} \begin{bmatrix} 0 & (1) & 2 & 3 \\ 0 & 2 & 4 & 6 \\ 8 & 8 & 8 & 8 \end{bmatrix} & \begin{bmatrix} 0 & (1) & 2 & 3 \\ 0 & 0 & 0 & 0 \\ 8 & 0 & -8 & -16 \end{bmatrix} & \begin{bmatrix} 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \\ (8) & 0 & -8 & -16 \end{bmatrix} & \begin{bmatrix} 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \\ (1) & 0 & -1 & -2 \end{bmatrix} \end{matrix}$$

To do these pivots in Scilab (provided 'P' has been loaded) enter:

$r=1; c=2; \text{execstr}(p)$ and $r=3; c=1; \text{execstr}(p)$

Moving the bottom row to the top and the middle row to the bottom gives the reduced row echelon form.

To immediately get the rref of a matrix a , enter: `rref(a)`

Continued on back.

- Find the basic and general solution to

$$2z = -4$$

$$x + 3y + 2z = 1$$

$$x + 3y + 3z = -1$$

Start with the augmented matrix. Reduce it to tableau or rref form. The basic variables are those with pivot points in their columns. Write them in terms of the parameters (nonbasics).

$$\begin{array}{ccc|c} x & y & z & \\ \hline 0 & 0 & 2 & -4 \\ 1 & 3 & 2 & 1 \\ 1 & 3 & 3 & -1 \end{array} \quad \text{augmented matrix} \quad \begin{array}{ccc|c} x & y & z & \\ \hline 0 & 0 & (1) & -2 \\ (1) & 3 & 0 & 5 \\ 0 & 0 & 0 & 0 \end{array} \quad \text{tableau}$$

Tableau system of equations: $z = -2$, $x + 3y = 5$, $0 = 0$

Basic variables: x, z . Parameter: y .

Basic solution: $x = 5$, $z = -2$, $y = 0$.

General solution: $x = 5 - 3y$, $z = -2$, y arb.