Math 414 Lecture 2

If the *rref* of an augmented matrix of a linear system were

x	y	z			
1	3	0	5	system	$x + 3y = 5$
0	0	1	–2	would be	$z = -2$
0	0	0	1		$0 = 1$

The reduced system would be
$$x + 3y = 5, \quad z = -2, \quad 0 = 1$$

The last equation is impossible, thus the answer is "no solutions".

A is *nonsingular* iff $\det(A) \neq 0$ iff A has an inverse.
A is *singular* otherwise.

Know the following definitions from the text or Math 311.
Let $v_1, \ldots, v_k$ be k vectors in an n-dimensional space.
A *linear combination* of the $v_1, \ldots, v_k$ is ... .
$v_1, \ldots, v_k$ is a *basis* iff ... .
The *dimension* of a subspace is ... .
In the *row-column* order of a matrix,
the first row's entries come first, then
the second row's entries, then
the third row's entries ... . This is SciLab's order.

If you haven't installed SciLab or something equivalent, do it before doing your homework. Google "SciLab". Then copy the file "414.txt" from www.math.hawaii.edu/414 to a file "414" on your computer.

Entering data

Enter:
```
a=[1 2 3]
a=[1; 2; 3]
a=[0 1 2; 3 4 5; 6 7 8]
a(2,2)
a(2,:)
a(:,2)
a
```

If you put two or more commands on the same line, separate them with ";". E.g.,
```
a(2,3)=5; a(3,2)=-1/3; r=2
```

To recover from errors, save *a* to a backup matrix *z*:
```
z=a
```
To recover *a* after it has been lost enter:
```
a=z
```

Pivoting

First pivot on the first column using row operations. Then pivot on the second column pivot by loading and using the *p* program. (It has to be loaded since it is a program I wrote and is not part of Scilab.)

Enter: `load('414')` Do this just once per session.

Enter:
```
r=2; c=2; execstr(p)
a=z
rref(a)
a
[a,eye(3)]
rref([a,eye(3,3)])
a=[1 2 3; 3 2 1; 1 1 1]
rref([a,eye(3,3)])
```

Getting rational answers i.e., ½ instead of .5

Enter:
```
a = [2 .5; 6 1/3]
[n,d] = rat(a)
```

inv(A) is the inverse of A. The most efficient way to find inv(A) by hand involves pivoting on the augmented matrix $[A \mid I] = [A, \text{eye}(n,n)]$ to get a matrix
  $\text{rref}([A, \text{eye}(n,n)]) = [I \mid B]$.
If there is an all 0 row, A is *singular* and has no inverse. Otherwise, inv(A) = B.

■ Find the inverse of A = $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ by pivoting.

[A, eye(2,2)] After 1st pivot After 2nd pivot
$$\begin{bmatrix} 1 & 2 & 1 & 0 \\ 3 & 4 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & -2 & -3 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & -2 & 1 \\ 0 & 1 & 1.5 & -.5 \end{bmatrix}$$

The last is in rref. Thus the inverse of B is $\begin{bmatrix} -2 & 1 \\ 3/2 & -1/2 \end{bmatrix}$

Saving data

To guard against losing your work in event of a crash, periodically save your work to your disk.

While in Scilab, to save your work to a file *hw3*, enter:
```
save('hw3')
```

To retrieve your work, start Scilab and enter:
```
load('hw3')
```

To load/save to a directory named c:\scilab enter:
```
save('c:\scilab\hw3')
load('c:\scilab\hw3')
```

Extra Credit

The first one to find an error in the lecture notes or homework gets an extra credit point. ½ point for misspellings.

Note *execstr(p)* always works on whatever is named '*a*'. $b = [1, 1; 2, 3]$; $r = 1$; $c = 1$; execstr(p) won't do the desired pivot on b.
  If you want to do the pivot on b, copy b to a, pivot on a then copy b,
```
a=b; r=1; c=1; execstr(p); b=a;
```
$c = a$. In Octave, use *eval(p)* instead of *execstr(p)*, in OldMatLab, use >p< .

Continued on back side .

Let $v_1, \ldots, v_k$ be row vectors.
Let $A = [v_1; v_k; \ldots; v_k]$ be the matrix with rows $v_1, \ldots, v_k$.

DEFINITION. rank(A) = the dimension of the subspace spanned by its rows.

LEMMA. Elementary row operations on a matrix change the rows but do not change the subspace spanned by the rows. The column rank of a matrix = its row rank.

COROLLARY. A and rref(A) have the same rank.

COROLLARY. rank(A) = the number of nonzero rows of rref(A).

THEOREM. If $v_1, \ldots, v_k$ are k vectors in an n-dimensional space and $A = [v_1; \ldots; v_k]$:

$$\begin{aligned}
v_1, \ldots, v_k \text{ independent} &\Rightarrow k \leq n \\
v_1, \ldots, v_k \text{ span the space} &\Rightarrow k \geq n \\
v_1, \ldots, v_k \text{ a basis} &\Rightarrow k = n \\
v_1, \ldots, v_k \text{ independent} &\Leftrightarrow \text{rank}(A) = k \\
v_1, \ldots, v_k \text{ span the space} &\Leftrightarrow \text{rank}(A) = n \\
v_1, \ldots, v_k \text{ is a basis} &\Leftrightarrow k = \text{rank}(A) = n
\end{aligned}$$