Math 414 Hw 4 Name ________________________________ Score____/ 16

Hw 91: 2, 4. Recommended 91: 1, 3, 5, 7.

The problems below are called *linear programming problems*. The inequalities are the *constraints*. The set S of points which satisfy the constraints is the set of *feasible solutions*. The function to be maximized or minimized is the *objective function*.

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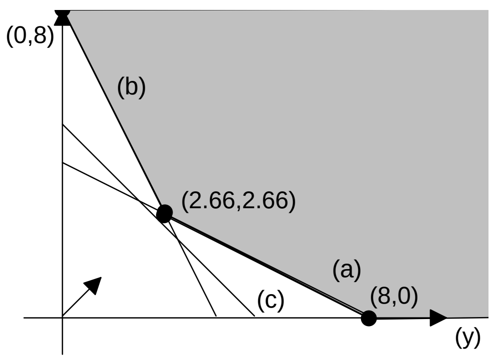
- Sketch the set of feasible solutions and, on the graph, give the coordinates of all extreme points and label the lines with their constraint numbers.
- Draw the gradient vector for the objective function. If the function is $ax+by$, the gradient vector is $[a,b]$. It points in the direction of maximum increase.
- Make a table listing the value of the objective function on each extreme point.
- Circle the maximum or minimum value rows. On the graph circle the extreme point or set of points which have this maximum or minimum. Label all boundary lines.

The first three problems have been done for you.

■ Minimize $z=x+y$ subject to the constraints:
a: $x+2y\geq 8$, $x\geq 0$,
b: $2x+y\geq 8$, $y\geq 0$,
c: $x+y \geq 5$.
Solution.



x	y	z
2.66	2.66	5.33
0	8	8
8	0	8

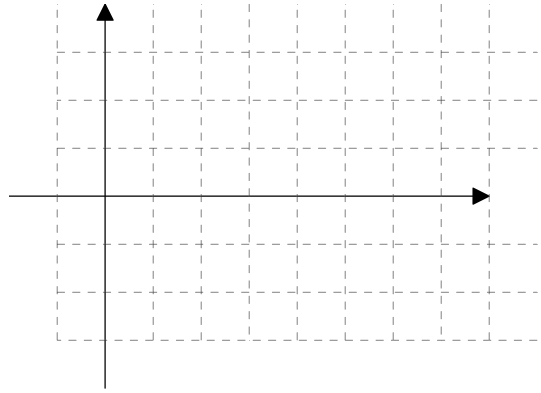
■ Maximize $z=x+y$ subject to the constraints:
a: $x+2y\geq 8$, $x\geq 0$,
b: $2x+y\geq 8$, $y\geq 0$,
c: $x+y \geq 5$.

Solution. the picture and table are the same as above but the region is unbounded in the direction of the gradient and the answer is "*no max*".

■ Minimize $z = x+y$ subject to the constraints:
a: $x+2y \geq 8$, $x\geq 0$,
b: $2x+y \geq 8$, $y\geq 0$,
c: $x+y \leq 5$.

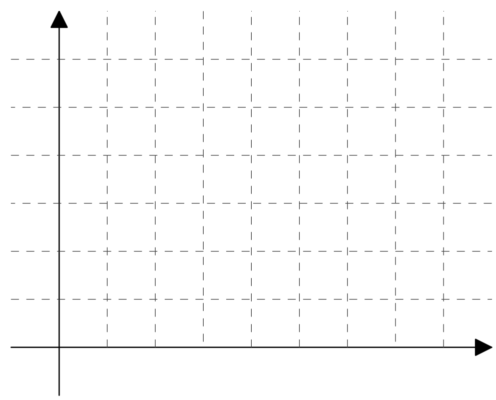
Solution. there are no points which satisfy all three constraints. Thus the answer is "*no feasible solutions*".

2′(6). **Minimize** $z=5x-3y$ *subject to the constraints*
a: $x+2y \geq 4$, $x\geq 0$,
b: $x+3y \leq 6$, $y\geq 0$.



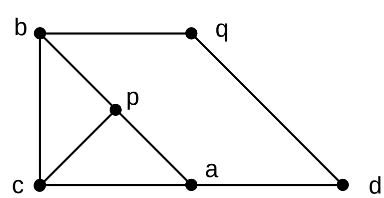
x	y	z

4(6). **Maximize** $z = 2x+3y$ *subject to the constraints*
a: $3x+y \leq 6$, $x\geq 0$,
b: $x+y \leq 4$, $y\geq 0$,
c: $x+2y \leq 6$.



x	y	z

Added problem(4). Suppose $f(x)$ is a linear function on a convex set S whose boundary is the quadrilateral with vertices c,b,q,d. Suppose $f(a)=1$ is a local maximum; $f(b)=-1$ is a local minimum. $\overline{ad}$, $\overline{ca}$ have the same length; $\overline{cp}$, $\overline{pa}$, $\overline{pb}$, have length 1. a,b,d,q are the vertices of a parallelogram. Study Lecture 4's two theorems; this is hard.



Then $f(c) =$ $f(d) =$ $f(p) =$ $f(q) =$