**Math 414 Lecture 5**

*Go to [www.math.hawaii.edu/~414](http://www.math.hawaii.edu/~414), click LPSolve. Install LPSolve. Run it on the simple example given.*

**Word problems**

Translate into general linear programming problems. Solve using Linsolve.

- **Golf carts are made in Detroit and Newark and shipped to dealers in Miami, Houston and LA.**

**Production:**
- Detroit can make 100/month,
- Newark can make 95/month.

**Needs:**
- Miami needs 60/month,
- Houston needs 60/month,
- LA needs 70/month.

**Shipping costs:**

<table>
<thead>
<tr>
<th></th>
<th>Miami</th>
<th>Houston</th>
<th>LA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Detroit</td>
<td>$50</td>
<td>$30</td>
<td>$70</td>
</tr>
<tr>
<td>Newark</td>
<td>$40</td>
<td>$60</td>
<td>$90</td>
</tr>
</tbody>
</table>

To minimize shipping costs, how many carts must be shipped from each plant (Detroit and Newark) to each dealer (Miami, Houston, LA)?

**Linear Programming Problem**

Let
- \( dm \) = # shipped from Detroit to Miami,
- \( dh \) = # Detroit to Houston,
- \( dl \) = # Detroit to LA,
- \( nm \) = # Newark to Miami,
- \( nh \) = # Newark to Houston,
- \( nl \) = # Newark to LA.

Minimize \( s = 50dm + 45p \)

with

\[
\begin{align*}
cl: & \quad 4l + 6p \leq 100 \\
gl: & \quad 10l + 12p \leq 100 \\
l: & \quad l + .5p \leq 12 \\
\end{align*}
\]

as opposed to

\[
\begin{align*}
cl: & \quad 4a + 6b \leq 100 \\
gl: & \quad 10a + 12b \leq 100 \\
l: & \quad a + .5b \leq 12 \\
\end{align*}
\]

To get LPSolve to calculate the slack for a constraint, convert the constraint to canonical form, e.g., convert \( dp: \) \( dm + dh + dl \leq 100 \) to \( dp: \) \( dm + dh + dl + dp = 100 \)

In LPSolve end all contraints with “;”.

LPSolve assumes variables are \( \geq 0 \), \( l, p \geq 0 \) isn’t needed.

For extra credit hand this in next time along with Hw 5.

Name _________________ Score ____/4
Now consider canonical problems: $AX = b$, $X \geq 0$.
We give formal proofs of theorems of Lecture 4.

**FACT.** Points sufficiently near a positive number are positive.

**THEOREM.** In the set $S$ of feasible solutions, a point is extreme iff it has more 0’s than nearby points.

**PROOF.** Suppose $p$ is an extreme point of $S$. We must show $p$ has more 0’s than nearby points. Assume for the sake of a contradiction that $p’$ is a nearby point with the as many 0’s as $p$. If $p’$ is sufficiently near $p$, then the positive entries of $p$ are also positive in $p’$ by the fact above. Thus $p’$ has 0’s in the same places as $p$. If we let $p''$ be a point collinear with $p$ and $p’$ and sufficiently near $p$ but on the other side from $p’$, then $p''$ also has 0 and positive entries in the same places as $p$ and also satisfies $AX = b$. The $p''$ is also a feasible solution. Thus $p$ is between two other feasible points. This contradicts the assumption that $p$ is extreme (by definition, extreme points are not between other points).

Suppose the matrix form of the canonical problem is maximize $z = C \cdot X$ again, $C \cdot X$ = the inner product.
where $A$ is the $m \times n$ coefficient matrix,
$X = [x_1; x_2; \ldots; x_n]$, $b = [b_1; b_2; \ldots; b_n]$, $C = [c_1; c_2; \ldots; c_n]$, $n = \text{number of variables}$, $m = \text{number of independent equations}$ = number of nonzero rows in the rref = number of basic variables. = number of independent columns (row rank = column rank).

$k = n - m = \text{number of parameters}.$

**LEMMA.** Every basic solution has $\leq m$ positive entries, $\geq k$ zeros.

**PROOF.** All the $k$ parameters are 0. Thus at least $k$ variables are 0. There are $m$ basic variables (they can be 0 or nonzero). Every nonzero variable is basic. Thus there are at most $m$ nonzero variables. □

**THEOREM.** $k = \text{the dimension of the set of solutions to } AX = b$.

**PROOF 1.** The space of all $X = [x_1, \ldots, x_n] \in \mathbb{R}^n$ has dimension $n$ and every time we add an independent equation, the dimension is reduced by 1. After adding $m$ equations the dimension of the set of solutions is $k = n - m$ which is the number of parameters.

**PROOF 2.** There are $k$ parameters $\Rightarrow$ there are $k$ coordinates which can vary independently $\Rightarrow$ there are $k$ dimensions.

**DEFINITION.** A set of $m$ variables is basic iff its associated set of columns is independent. □