

Page 99. *Start on the other side.*

1'(5). Suppose the constraint matrix A is $[2\ 2\ 1\ 1\ 6; 1\ 0\ 0\ 1\ 2; 1\ 1\ 1\ 0\ 4]$ and the constant vector b is $[5; 0; 3]$. Classify the following possible solutions of $AX=b$ and $X \geq 0$. Circle the one *best*, most specific, answer.

Solution if $AX=b$, feasible if $X \geq 0$, basic solution = a solution whose nonzero variables have independent columns (but not necessarily identity columns) in A .

- $[0; 3; 0; 5; 6]$ ——— none, feasible, solution, basic solution, feasible solution, basic feasible solution
- $[0; 2; 1; 0; 0]$ ——— none, feasible, solution, basic solution, feasible solution, basic feasible solution
- $[-1; 2; 2; 1; 0]$ ——— none, feasible, solution, basic solution, feasible solution, basic feasible solution
- $[2; 2; 3; -1; 0]$ ——— none, feasible, solution, basic solution, feasible solution, basic feasible solution
- $[1; 2; 0; -1; 0]$ ——— none, feasible, solution, basic solution, feasible solution, basic feasible solution

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Maximize $p = 7.50(ah+ch+sh+ih) +$ _____

Simplified, this becomes -

$$p = .3ah + 3.15ch +$$

Constraints (6 inequalities, 4 equalities)

1: $ah + al \leq 800$ 2: $ch + cl \leq 600$ 3: _____ 4: _____

5: $(5ah + 6.5ch + 4sh + 18ih)/(ah+ch+sh+ih) = 7$

Don't replace the denominator by 1300. Replacing the denominator by 1300 is part of the solution, not part stating the problem.

Multiplied out this gives $5ah+6.5ch+4sh+18ih=7(ah+ch+sh+ih)$ which simplifies to --

$$-2ah - .5ch - 3sh + 11ih = 0$$

6: _____
simplified

7: _____
simplified

8: _____
simplified

9: $ah + ch + sh + ih =$ _____

10: _____

$$ah, ch, sh, ih, al, cl, sl, il \geq 0$$

Solution. ah has been done for you, I've filled in the decimal parts of al, ch, cl, sh, ih, il (if you think one is wrong, cross it out). You must fill in the integer parts.

Variables	
$ah=$ 565.297	$al=$.425
$ch=$.219	$cl=$.781
$sh=$.759	$sl=$
$ih=$.725	$il=$.795
Objective	$p=$.587

HW 8 (start here)

NOTE. To determine if say columns 1,3 and 4 of a matrix A are independent — check that $\text{rank}(A(:,[1,3,4]))=3$. If the rank function crashes.

The theorems of Lecture 8 are helpful for problem 16.

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1'(5). Suppose the canonical constraint matrix A and the constant vector are:

$A=$

2	2	1	1	6
1	0	0	1	2
1	1	1	0	4

$b=$

5
0
3

On the answer page, give the best classification of each possible solution of $AX=b$ and $X \geq 0$.

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10(16).

After making the corrections below, translate the word problem into a general linear programming problem.

CORRECTIONS. High-octane gas has a higher price than low-octane gas.

In the Revenue column, high-octane should be 7.50 and low-octane should be 6.50.

Finally, the problem as stated is inconsistent and has no solution since the supplies are not adequate to meet the demands.

To correct this, increase the supply of Alkylate from 700 to 800. This has been done in constraint 1: .

NOTE. The vapor pressure and demand must be met exactly; the octane ratings must be \geq the stated amount.

The volumes are in barrels, the revenue and cost are in dollars per barrel.

VARIABLES. Let p be the profit (total profit, not profit per barrel).

Let ah, ch, sh, ih be the volume of Alkylate, Catalytic cracked, Straight-run and Isopentane used to make the 1300 barrels of high-octane gas.

Let al, cl, sl, il be the volume of Alkylate, Catalytic cracked, Straight-run and Isopentane used to make the 800 barrels of low-octane gas.

RECOMMENDATION. Enter the problem in a text processor such as Notepad, then run it on Linsolve.

Three constraints have been done.

The objective function $p = \dots$ has been started, finish filling it out.

State the objective function and constraints 6, 7 and 8 two ways. First state them as unsimplified possibly nonlinear constraints that most closely match the statement of the problem. Then simplify them to linear constraints acceptable to Linsolve (which handles decimals but not fractions).

Constraint 5: has been done both ways as an example.

Warning, in ambiguous word problems what sounds like $=$ often mean \leq or \geq .

“10 tons of coal is required” is best translated as “ $c \geq 10$ ”, not “ $=10$ ”.

“Can produce 10 tons of coal” means “ $c \leq 10$ ”.