

Complete both sides. Recommended 150: 1a, 3a, 5, 9, 21.

Write the phase one problem for the given original problem using the phase one objective. Let r', s' be the extra variables.

Write the matrix for this problem. Get Tableau 1 by pivoting on the identity columns. **Circle** all pivots.

If the first phase fails or if it produces a nonzero optimum, write "Empty region, no feasible solutions."

If the first phase produces a minimum of 0, but an extra variable is basic, pivot, if possible, on a nonzero entry of its row to make some other variable basic and make it a parameter.

Phase two begins with the basic feasible solution produced by phase one. The initial matrix for phase two is the matrix for the last tableau of phase one with the nonbasic extra variable columns deleted and the phase one objective row replaced by the original objective row. Convert this matrix to a tableau by pivoting to make identity columns. This is tableau 1. Run the simplex method on this matrix.

Recommendation: use Matlab to get the tableaus, then check your answer using LinSolve. In Linsolve, enter the original problem with the original objective and without the extra variables. In the output, r', s' and e will be *Extra1*, *Extra2*, and *Extra*.

The constraints for 1, 2, 3 are equalities and thus have no slacks and in fact have at most one solution. After running the two-phase simplex algorithm, check your work by solving the system of equations using rref(a).

1(7) ORIGINAL PROBLEM

$$\text{Max } z = 2x + y$$

with

$$r: x + y = 2$$

$$s: 4x - y = 2$$

$$x, y \geq 0$$

PHASE ONE PROBLEM

$$\text{Max } e =$$

Phase one matrix

| | x | y | r' | s' | b |
|-----|-----|-----|------|------|-----|
| | | | | | |
| e | | | 1 | | |

Tableau 1

| | x | y | r' | s' | b |
|-----|-----|-----|------|------|-----|
| | | | | | |
| e | -5 | | | | |

Tableau 2

| | x | y | r' | s' | b |
|-----|-----|------|------|------|-----|
| | | | | | |
| e | | -5/4 | | | |

Last tableau

| | x | y | r' | s' | b |
|--|-----|-----|------|------|-----|
|--|-----|-----|------|------|-----|

| | | | | | |
|-----|--|--|-----|--|--|
| | | | 4/5 | | |
| | | | | | |
| e | | | | | |

Phase 2 matrix

| | x | y | b |
|-----|-----|-----|-----|
| | | | |
| z | -2 | | |

Pivot to make tableau 1. Note change from e to z .

Tableau 1

| | x | y | b |
|-----|-----|-----|-----|
| | | | |
| z | | | |

Entries should total to 34/5

$$\text{Max } z = \text{_____ when } x = \text{_____, } y = \text{_____}.$$

Warning, to run the two-phase algorithm, you must make the constant column positive.

2(5) ORIGINAL PROBLEM

$$\text{Max } z = 2x + y$$

with

$$r: x + y = -2$$

$$s: 4x - y = 2$$

$$x, y \geq 0$$

PHASE ONE PROBLEM

$$\text{Max } e =$$

Phase one matrix

| | x | y | r' | s' | b |
|-----|-----|-----|------|------|-----|
| | | | | | |
| e | | 0 | | | |

Tableau 1

| | x | y | r' | s' | b |
|-----|-----|-----|------|------|-----|
| | | | | | |
| e | | 2 | | | |

Last tableau

| | x | y | r' | s' | b |
|-----|-----|-----|------|------|-----|
| | | | | 1/4 | |
| e | | 5/4 | | | |

Answer

Complete both sides

3(5) ORIGINAL PROBLEM

PHASE ONE PROBLEM

Max $z = 2x + y$

with

$r: x + y = -2$

$s: 4x - y = -2$

$x, y \geq 0$

Phase 1 matrix

| | x | y | r' | s' | b |
|-----|-----|-----|------|------|-----|
| | | | | | |
| e | 0 | | | | |

Tableau 1

| | x | y | r' | s' | b |
|-----|-----|-----|------|------|-----|
| | | | | | |
| e | 5 | | | | |

Check entries should be digits which total to 2.

Answer

4(3) Convert the following to a canonical problem with $b \geq 0$, then to a phase one problem. Don't continue with the solution. Note that min w becomes max $-w$.

If you need both a slack and an extra variable s , let s' be the extra and let s be the slack.

ORIGINAL PROBLEM

Min $w = 8x + 6y + 11z$

with

$r: 5x + y + 3z \leq 4$

$s: 5x + y + 3z \geq 2$

$t: -2x - 4y - 7z = -5$

$x, y, z \geq 0$.

CANONICAL PROBLEM WITH $b \geq 0$.

Max $-w =$

with

$r:$

$s:$

$t:$

$x, y, z, _, _ \geq 0$

CANONICAL PROBLEM WITH EXTRA VARIABLES

Should be only 2 extra variables.

PHASE ONE PROBLEM

Complete both sides