Phase two case in which an extra variable is basic

Recall: you may delete phase one columns which belong to extra variables which are parameters. But if an extra variable is basic, ignore the “pick most negative objective coefficient” rule. Instead pivot on any other nonzero coefficient in the the extra variable's row. After pivoting, the extra basic variable departs and you may delete its column.

If you need the dual solution keep the extra variable columns but don’t pivot on their entries. Ignore their objective values even if negative. The dual variable signs are determined separately.

Previously, we pivoted on the initial phase 1 and phase 2 matrices to get the initial tableaus. These pivots changed only the objective row. Now, instead of pivoting, you can use the Objective Row Theorem to get the initial objective row directly.

**Tableau 1, phase 1**

**Tableau 2, phase 1**

**Tableau 3, phase 1**

**Tableau 1, phase 2**

**Tableau 2, phase 2**

Primal solution.

min \( z = -3 \), \( x = 0, y = 3, u = 0 \) with slacks \( r = 0, s = 0, t = 2 \).

Dual solution.

max \( z = -3 \), \( r = 2, s = 1, t = 0 \) slacks \( x = 0, y = 0, u = 0 \).

**Factors which change a dual variable’s sign:**
- Changing max to min.
- Multiplying a primal constraint by -1.
- Subtracting a slack instead of adding.
Multiple optimal solutions

The final tableau for each problem is below.
Either (1) circle “unique optimal solution” or (2) find a second optimal solution.

Problems 1, 2, 3 all have the following constraints

\[ r: x+y \leq 3 \]
\[ s: x \leq 2 \]
\[ x, y \geq 0 \]

1. \( \text{max } z = y \)

Final tableau.

<table>
<thead>
<tr>
<th></th>
<th>x</th>
<th>y</th>
<th>r</th>
<th>s</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>s</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>z</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>3</td>
</tr>
</tbody>
</table>

**FIRST OPTIMAL SOLUTION:** \( \text{max } z = 3, x = 0, y = 3 \).

**SECOND OPTIMAL SOLUTION**

unique

optimal

solution

max \( z = ____ \)

\( x = ____ \)

\( y = ____ \)

2. \( \text{max } z = x+y \)

Final tableau.

<table>
<thead>
<tr>
<th></th>
<th>x</th>
<th>y</th>
<th>r</th>
<th>s</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>x</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>z</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>3</td>
</tr>
</tbody>
</table>

**FIRST OPTIMAL SOLUTION:** \( \text{max } z = 3, x = 2, y = 1 \).

**SECOND OPTIMAL SOLUTION**

unique

optimal

solution

max \( z = ____ \)

\( x = ____ \)

\( y = ____ \)

3. \( \text{max } z = x \)

\[ x \quad y \quad r \quad s \quad b \]
<table>
<thead>
<tr>
<th></th>
<th>y</th>
<th>0</th>
<th>1</th>
<th>1</th>
<th>-1</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>z</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

**FIRST OPTIMAL SOLUTION:** \( \text{max } z = 2, x = 2, y = 1 \).

**SECOND OPTIMAL SOLUTION**

unique

optimal

solution

max \( z = ____ \)

\( x = ____ \)

\( y = ____ \)

**THEOREM.** If all parameters have positive objective row coefficients, the optimal solution is unique.

If the objective coefficient of a final-tableau parameter is 0, and its least \( \theta \)-ratio is positive select it as an entering variable to get a different optimal basic solution.

**Problems with secondary objectives**

Suppose multiple solutions are optimal with respect to a *primary* objective function. From these, we wish to select those which are optimal with respect to some *secondary* objective function.

- A business man may wish to minimize costs (primary objective) and, among all solutions which minimize costs, he may want the one which requires the least amount of debt (secondary objective).

**SOLUTION.** Solve the primary problem, ignoring the secondary objective. Suppose the minimum cost is \( C \). Add “primary objective = \( C \)” as a new constraint and solve the problem with the secondary objective. The new constraint guarantees that the solution remains optimal with respect to the primary objective. It also makes some old constraint redundant, you may delete the redundant old one. LPSolve works whether it is deleted or not.

- Primary objective: \( \text{max } z = x+y \)

Secondary objective: \( \text{min } w = y \)

**PRIMARY PROBLEM.**

\[ \text{max } z = x+y \]

with See the figure on the left.

\[ r: x+y \leq 3 \]
\[ s: x \leq 2 \]
\[ x, y \geq 0 \]

**Primary solution**

\( \text{max } z=3, x=0, y=3 \).

**SECONDARY PROBLEM.**

\[ \text{min } w = y \]

\[ t: x+y = 3 \] ← New constraint.
\[ r: x+y \leq 3 \] ← This becomes redundant.

\[ s: x \leq 2 \]

Delete it before running simplex method.

\( x, y \geq 0 \)

Final solution: \( \text{max } z = 3, \text{ min } w = 1, x = 2, y = 1 \).