Math 414 Lecture 21

Multiple optimal solutions

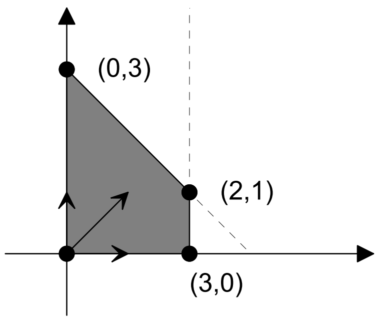
The final tableau for each problem is below. Either circle "unique optimal solution" or find a second optimal solution.

Problems 1, 2, 3 all have the following constraints

r: $x+y \le 3$

s: $x \le 2$

$x, y \ge 0$



1. max $z = y$
final tableau.

	x	y	r	s	b
y	1	1	1	0	3
s	1	0	0	1	2
z	1	0	1	0	3

FIRST OPTIMAL SOLUTION: max $z=3$, $x=0$, $y=3$.

second optimal solution

unique optimal solution — max $z =$ ____

$x =$ _____

$y =$ _____

2. max $z = x+y$

	x	y	r	s	b
y	0	1	1	-1	1
x	1	0	0	1	2
z	0	0	1	0	3

FIRST OPTIMAL SOLUTION: max $z=3$, $x=2$, $y=1$.

second optimal solution

unique optimal solution — max $z =$ ____

$x =$ _____

$y =$ _____

3. max $z = x$

	x	y	r	s	b
y	0	1	1	-1	1
x	1	0	0	1	2
	0	0	0	1	2

FIRST OPTIMAL SOLUTION: max $z=2$, $x=2$, $y=1$.

second optimal solution

unique optimal solution — max $z =$ ____

$x =$ _____

$y =$ _____

THEOREM. If the objective coefficient of a final-tableau parameter is 0, select it as an entering variable to get a possibly different optimal basic solution (to be genuinely different some value must change, not just the set of basic variables). If all parameters have positive objective row coefficients, the optimal solution is unique.

Problems with secondary objectives

Suppose multiple solutions are optimal with respect to a *primary* objective function. From these, we wish to select those which are optimal with respect to some *secondary* objective function.

- A business man may wish to minimize costs (primary objective) and, among all solutions which minimize costs, he may want the one which requires the least amount of debt (secondary objective).

SOLUTION. Solve the primary problem, ignoring the secondary objective. Suppose the minimum cost is C. Add "primary objective$=C$" as a new constraint and solve the problem with the secondary objective. The new constraint guarantees that the solution remains optimal with respect to the primary objective. The new constraint makes an old one redundant, delete the redundant old one.

- Primary objective: max $z = x+y$

Secondary objective: min $w = y$

PRIMARY PROBLEM.

max $z = x+y$

with — See the figure on the left.

r: $x+y \le 3$

s: $x \le 2$ — Primary solution

$x, y \ge 0$ — max $z=3$, $x=0$, $y=3$.

SECONDARY PROBLEM:

min $w=y$

r: $x+y \le 3$ ← This becomes redundant. Delete it before

s: $x \le 2$ running LinSolve. Linsolve doesn't know how

t: $x+y = 3$ to delete a basic extra variable.

$x, y \ge 0$

Final solution: max $z = 3$, min $w = 1$, $x = 2$, $y = 1$.