

Math 414 Lecture 23 Exam 3 week from today

Recall: CONSTANT COLUMN THEOREM. Place the initial basic identity columns at the end to make an identity matrix. Let T = the matrix in that region of the final tableau. Let B = original constant column (exclude the objective value).

Assume the constant column entries are ≥ 0 .
Let B' = the final constant column, then
 $T \cdot B = B'$.

■ Initial matrix

	x	y	s'	r	s	b
r	1	-2	0	1	0	10
s	1	1	-1	0	1	5
z	-2	-1	0	0	0	0

Complete the final tableau

	x	y	s'	r	s	b'
r	3	0	-2	1	2	
y	1	1	-1	0	1	
z						

Original $B = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$, final $B' = T \cdot B = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 5 \end{bmatrix} = \begin{bmatrix} 20 \\ 5 \end{bmatrix}$.

Constant column sensitivity

Identify a "solution" with the constraint boundaries on which it lies (the constraints with 0 slacks). Thus a solution will move with its constraint lines.

Let b_r = the right-hand constant of constraint r .
When this constant changes, the constraint boundary moves. So does any optimal solution on the boundary. It remains optimal (or better) but may become infeasible.

Let $B(b_r \rightarrow p)$ be the original constant column B with b_r replaced by a new variable p . By the Constant Column Theorem, the final constant column $B' = T \cdot B(b_r \rightarrow p)$. This expresses the final solution as a function of p and allows us to determine how the solution changes with respect to p .

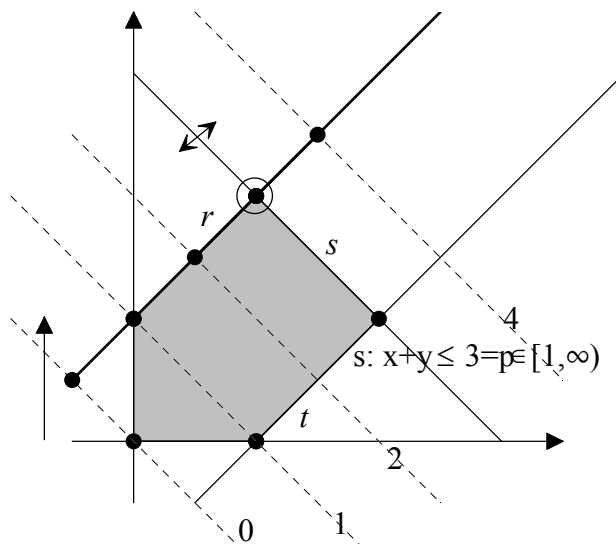
The range of values of b_r for which the solution is feasible is the set of p such that $B' \geq 0$.

■ Primal problem

$\max z = y$
with

- $r: -x + y \leq 1 \leftarrow b_r$
- $s: x + y \leq 3 \leftarrow b_s$
- $t: x - y \leq 1 \leftarrow b_t$
- $x, y \leq 0$

■ What happens when $b_s = 3$ changes?



Initial matrix

	x	y	r	s	t	b	$B(b_s \rightarrow p)$
r	-1	1	1	0	0	1	$\rightarrow 1$
s	1	1	0	1	0	3	$\rightarrow p$
t	1	-1	0	0	1	1	$\rightarrow 1$
	0	-1	0	0	0	0	

Complete the final tableau.

	x	y	r	s	t	b'	B'
	0	1	0	0	0	0	0
$1 y$	0	1	0.5	0.5	0	2	$\rightarrow .5 + .5p$
$0 x$	1	0	-0.5	0.5	0	1	$\rightarrow -.5 + .5p$
$0 t$	0	0	1	0	1	2	$\rightarrow 2$
						2	$\rightarrow .5 + .5p$

Thus T is the shaded matrix in the final tableau and $B(b_s \rightarrow p) = \langle 1; p; 1 \rangle$, $B' = \langle .5 + .5p; -.5 + .5p; 2 \rangle$.

This is feasible

- iff $.5 + .5p \geq 0$ and $-.5 + .5p \geq 0$ and $2 \geq 0$
- iff $1 + p \geq 0$ and $-1 + p \geq 0$
- iff $p \geq -1$ and $p \geq 1$
- iff $p \geq 1$ iff $p \in [1, \infty)$. Write the answer as an interval.

Answer. If $b_s = 3$ is replaced by a variable p , the optimal solution as a function of $b_s = p$ is

$\max z = .5 + .5p$
at $x = -.5 + .5p$
 $y = .5 + .5p$ for $b_s = p \in [1, \infty)$.

- Is the solution still feasible if $b_s = 3$ changes to 0?
- What is the optimal solution if $b_s = 3$ is changed to 6?