Math 414 Lecture 24 Included in Friday's exam

Objective coefficient sensitivity

DEFINITION. For any primal variable x, let c_x be the coefficient of x in the objective function.

- If the objective is min $w = 3x - 4y - 2z$, then $c_x = 3$, $c_y = -4$, $c_z = -2$

If one of these coefficients, say c_x changes, what happens to the optimal solution?

The objective gradient vector changes but the contraints do not. Thus the feasible region is unchanged. Feasible and extreme solutions do not move and do not become unfeasible. However optimal solutions may cease to be optimal.

As the objective function rotates, the optimal solution may move from one extreme to an adjacent extreme. At the point of changeover, the line segment between the adjacent extremes becomes optimal and the objective vector is perpendicular to the line segment. Mark these changeover directions, by drawing an outward pointing perpendicular vector from each line segment. Translate these *boundary-segment perpendiculars* to the origin.

- Draw dotted rays from the boundary-segment perpendiculars to mark off the pie-shaped segments of objective coefficient vectors for which each is optimal.

Primal problem.

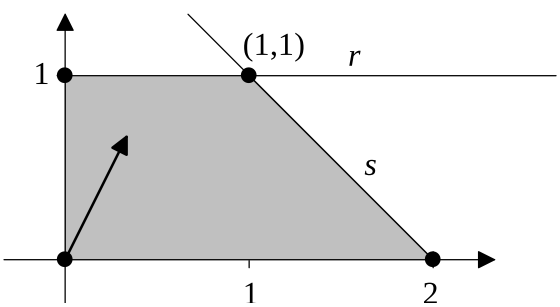
max $z = 1x + 2y$

with

r: $y \le 1$

s: $x + y \le 2$

$x, y \ge 0$



Then $c_x = 1$, $c_y = 2$ and the final tableau is

	x	y	r	s	b
y	0	1	1	0	1
x	1	0	-1	1	1
z	0	0	1	1	3

Optimal solution. max $z = 3$ at $x = 1$, $y = 1$.

- Over what range (i.e., interval) of values of c_x is this solution optimal?

To determine this range of values, replace the coefficient with a variable, say p, and recalculate the objective row with the Objective Row Theorem.

The range of values for which the solution is optimal is the set of p such that the objective row coefficients (exclude the unshaded objective value entry) are ≥ 0.

		p	2	0	0	0
		x	y	r	s	b
2	y	0	1	1	0	1
p	x	1	0	-1	1	1
	z	0	0	$2-p$	p	$p+2$

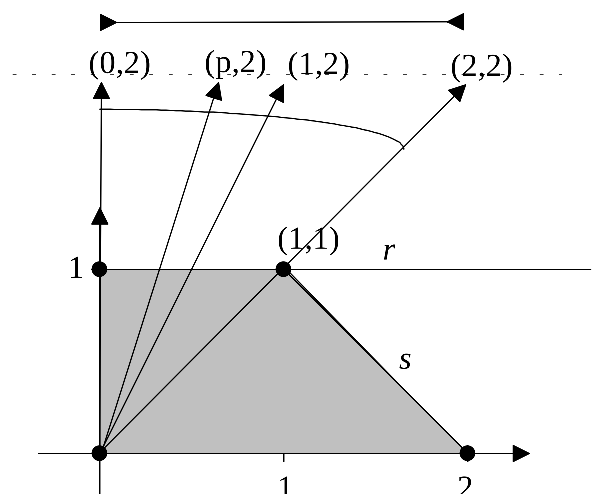
The solution is optimal

iff $2 - p \ge 0$ and $p \ge 0$

iff $2 \ge p$ and $p \ge 0$

iff $p \in [0, 2]$.

Picture. $p \in$ [0-------------------------------2]



Answer. If $c_x = 1$ is replaced by the variable p, the solution max $z = 3$ at $x = 1$ and $y = 1$ is optimal for max $z = px + 2y$ for $\boldsymbol{p \in [0, 2]}$.

- What is the range for $c_y = 2$ such that $(1, 1)$ remains optimal?

		1	p	0	0	0
		x	y	r	s	b
p	y	0	1	1	0	1
1	x	1	0	-1	1	1
	z					

The solution is optimal

iff

iff

Answer. The range is $c_y = p \in$