Math 414 Lecture 30

The greedy algorithm provides the initial transportation matrix.

matrix	W_1	W_2	W_3	W_4	Supply
P_1	1 / [30]	4 / 0	5 / 0	3 / [10]	40
P_2	8 / 0	6 / 0	1 / [40]	1 / [10]	50
P_3	6 / 0	7 / [20]	1 / 0	3 / [30]	50
Demand	30	20	40	50	

We shall successively improve it to an optimal solution.

The x_{ij}'s with circled values are the initial basic variables. Copy them to the initial transportation tableau and circle them. The ij^{th} dual constraint with slacks is x_{ij}: $p+w_j+o_{ij} = c_{ij}$.

In a simplex tableau, the objective row entry below the variable x_{ij} = the dual slack = $o_{ij} = c_{ij} - p_i - w_j$.

Loop: For each circled basic variable, the simplex objective row entry is $o_{ij} = 0$. Thus $p_i + w_j + o_{ij} = c_{ij}$ becomes $p_i + w_j = c_{ij}$.

Use these equations to solve for the dual variables $p_1, p_2, p_3, w_1, ..., w_4$. There are 7 dual variables but only 6 equations, hence one must be set arbitrarily. Choose p_1 and set it to 0. Use the 6 equations $p_i + w_j = c_{ij}$ to solve for the remaining 6 dual variables $p_2, p_3, w_1, ..., w_4$. Write the p_i's on the right border; the w_j's on the bottom border.

tableau	W_1	W_2	W_3	W_4	p_i
P_1	1 / [30]	4 / -3	5 / 2	3 / [10]	0
P_2	8 / 9	6 / 1	1 / [40]	1 / [10]	-2
P_3	6 / 5	7 / [20]	1 / -2	3 / [30]	0
w_j	1	7	3	3	

For parameters, $x_{ij}=0$. So instead of writing x_{ij} in the square, write the objective row entry o_{ij} which is the dual slack $o_{ij} = c_{ij} - p_i - w_j$.

If no objective coefficient is negative, stop, you have an optimal solution. Otherwise —

Choose the next entering basic variable as usual:

Pick the parameter x_{ij} with the most negative objective (uncircled) row entry o_{ij}. Erase o_{ij}; shade its square; add a circle.

In row i, the circled x_{ij} values must sum to s_i, i.e., $\Sigma_j x_{ij} = s_i$.

In column j, the circled values must sum to d_j.

Thus if one circled x_{ij} value in a row or column increases (decreases), some other value must decrease (increase).

Chase these increases and decreases around until you find a loop back to the entering variable. This loop begins and ends at the shaded entering variable; its other corners are the circled basic variables whose values must increase or decrease.

Alternately mark the corners of the loop "+" or "−" starting with a "+" on the entering variable. We will increase the "+" values; decrease the "−" ones.

How much can be added and subtracted from the loop variables? The − values cannot decrease below 0 since $x_{ij} \geq 0$. The maximum amount of change = the minimum of the values labeled "−".

Add this to the "+" variables; subtract it from the "−" variables.

The "−" variable with the minumum value goes to 0; it (pick one if two or more go to 0) is the departing basic variable. Remove the square's circled basic variable.

Repeat the loop (keep the x_{ij}'s; recalculate all p_i's, w_j's, and o_{ij}'s).

Picking the most negative o_{ij} equals selecting a simplex column with the most negative objective coefficient. Picking the first basic variable which goes to 0 equals picking the variable with the minimum θ ratio to be the departing variable.

Transportation algorithm

Start with an initial transportation matrix (greedy algorithm).
Repeat until an optimal solution is found:

- Set $p_1 = 0$.
- For the remaining p_i, w_j,
  calculate p_i, w_j using $p_i + w_j = c_{ij}$ for each circled basic x_{ij}.
  calculate o_{ij} using $o_{ij} = c_{ij} - p_i - w_j$ for each parameter x_{ij}.
- If no objective coefficient (uncircled o_{ij}) is negative, stop. You have an optimal solution. Otherwise:
- Pick the variable with the most negative o_{ij} as the entering variable. Erase o_{ij}; shade the square; add a circle.
- Find a loop, starting with the entering variable and whose other corners are basic (circled). Alternately mark the corners "+" and "-", starting with a "+" on the entering variable.
- Increase the "+" and decrease the "-" corner by the minimum of the "-" values.
- Remove the circled departing variable (a minimum "-" value) which went to 0.

	W_1	W_2	W_3	W_4	p_i
P_1	1 / [30]	4 + / [ ] (shaded)	5 / 2	3 − / [10]	0
P_2	8 / 9	6 / 1	1 / [40]	1 / [10]	-2
P_3	6 / 5	7 − / [20]	1 / -2	3 + / [30]	0
w_j	1	7	3	3	

	W_1	W_2	W_3	W_4	p_i
P_1	1 / [30]	4 / [10]	5 / 5	3 / 3	0
P_2	8 / 6	6 / 1	1 − / [40]	1 + / [10]	1
P_3	6 / 2	7 / [10]	1 + / [ ] (shaded)	3 − / [40]	3
w_j	1	4	0	0	

	W_1	W_2	W_3	W_4	p_i
P_1	1 / [30]	4 / [10]	5 / 7	3 / 3	0
P_2	8 / 6	6 / 1	1 / 2	1 / [50]	1
P_3	6 / 2	7 / [10]	1 / [40]	3 / [0]	3
w_j	1	4	-2	0	

Solution: min $z = 1\cdot30 + 4\cdot10 + 1\cdot50 + 7\cdot10 + 1\cdot40 + 3\cdot0 = 230$
at $x_{11}=30$, $x_{12}=10$, $x_{24}=50$, $x_{32}=10$, $x_{33}=40$, $x_{34}=0$, the rest$=0$