Math 414 Practice Exam 5 Don’t turn in.
This includes previous problems from Practice Exam 3 for comparison purposes. One or more of these might be on the final.

1(5) There are 9 grain storage areas: \(a, b, c, d, e, f, g, h, i\). 
\(a-3-c\) means you can directly ship 3 tons of grain from \(a\) to \(c\) or to \(a\) for $30 dollars.

\[ a-1-b, a-3-c, a-4-d, b-2-d, b-3-e, c-2-d, c-1-f, \]
\[ d-1-e, e-2-g, e-4-i, f-2-g, f-2-h, g-1-i, h-2-i. \]

Find the minimum shipping cost (in $10’s) from each node below to node \(a\).

\[ d \rightarrow a \_\_\_, e \rightarrow a \_\_\_, f \rightarrow a \_\_\_, g \rightarrow a \_\_, h \rightarrow a \_\_, i \rightarrow a \_\_. \]

1 point for the answer, 1 point for the graph, 3 points for correctly running the algorithm.

2(8) There are 9 grain storage areas: \(a, b, c, d, e, f, g, h, i\).
\(a-3-c\) means you can directly ship 3 tons of grain from \(a\) to \(c\).

\[ a-2-b, a-6-c, a-6-d, b-4-e, c-4-d, c-4-f, d-2-b, d-6-e, e-3-g, e-4-i, f-3-g, f-3-h, g-6-i, h-2-i. \]

Find the maximum amount of grain which can be shipped from \(a\) to \(i\)?

1 point for the answer, 2 points for drawing the graph and listing how much should be shipped along each edge. 5 points for correctly running the algorithm.

3(7) A bowl with 20 cups of soup is passed among some of the 9 people \(a, b, c, d, e, f, g, h, i\). It starts with “\(a\)” and stops at “\(i\)”. “\(a,2 \rightarrow b, c, d\)” means “\(a\)” takes 2 cups of soup from the bowl and then passes the bowl to one of \(b, c, d\).

\[ a,2 \rightarrow b, c, d \]
\[ b,1 \rightarrow e \]
\[ c,2 \rightarrow d, f \]
\[ d,1 \rightarrow b, e \]
\[ e,1 \rightarrow g, i \]
\[ f,2 \rightarrow g, h \]
\[ g,1 \rightarrow i \]
\[ h,1 \rightarrow i \]

Find the minimum amount of soup which might be left in the bowl when it reaches \(i\).

1 point for the answer, 4 points for modeling the problem as an assignment, max flow, shortest or longest path problem. 2 points for using the model to solve it.

4(14) Modular homes are made in plants \(P, Q, R, S, T\) and then shipped to housing sites \(A, B, C, D, E\). Each plant makes one house and each site gets one house. The shipping costs from a plant to a site are given in the matrix.

Which plants should ship to which sites in order to minimize shipping costs? Find the minimum shipping cost.

![Matrix](image)

What is the minimum shipping cost?

How many nails should be shipped from each plant to each housing site?
2 points for the answer. 4 points for correctly modeling the problem. 10 points for showing the steps of the appropriate algorithm.

5(16) Nails are made in plants \(P, Q, R, S\) and then shipped to housing sites \(A, B, C, D\). The shipping costs from a plant to a site are given in the matrix. Also given are the amounts of nails (in bushels) produced by the plants and needed by the housing sites. (From Practice Exam 3).

![Matrix](image)

What is the minimum shipping cost?

Players I and II choose heads \(H\) or tails \(T\). If both choose \(H\), I wins $2, if both choose \(T\), I wins $4. If they choose different sides, II wins $3.

\[ H \quad P \]
\[ T \quad 1-P \]

\[ pH + (1-p)T \]

Fill in the payoff matrix.

Find an optimal mixed strategy for player I.

What can I expect to win on average with this strategy?
Math 414  Review 5

Final: Friday, Dec. 16, 12:00 - 2:00 in our usual classroom.

Material.
The final is cumulative. It will have about 100 points, twice the usual 50 points. It will consist entirely of problems; there will be no proofs, no statements of theorems or definitions, no thought problems. Unlike the previous exams, you will put your answers and work on a separate sheet.
You won’t get homework credit for doing the practice exam but one or more of the practice exam problems will probably be on the final.

The review below covers only the material since the last exam. This will constitute less than half of the exam. The final is cumulative.

Definitions and rules. Understand the meaning, statements won’t be required.
Directed and undirected graphs, node, edge, flow, volume, cut, capacity, network, source or origin, sink, shortest path, longest path.
Acyclic graph, early/late job times, critical node.
Two-person zero-sum game. Payoff matrix, strategy, maximin and mixed strategies.

Be able to do
All previous problems plus the following which will be somewhat emphasized since they have not appeared on an earlier exam:
Assignment, Max-flow, shortest path, and longest path problems. Be able to run the algorithms for these problems.
Be able to setup marriage (matching) problems, equipment-replacement problems and job-scheduling problems.
Find optimal mixed strategies for two-person zero-sum games.

Suggested Exercises.
All homework exercises and exercises of past reviews plus
364: 3, 5, 7abc.
375: 1, 3, 5.
385: 1, 7.