Math 414 Practice Exam 1 Hw ___/50

Sit two apart during the exam. In case of a bomb threat, the exam will be held on the steps of Hamilton Library, or, if it's raining, in front of Kennedy theatre. Develop enough proficiency to do this in 52 minutes. Homework credit given for problems 5-14 if turned in on exam day. Bring a calculator and scratch paper.

1(0) Definitions. For $x, x_1, x_2, \ldots, x_n \in R^n$ and $S \subseteq R^n$, for A a matrix,

(a) x is an *extreme point* of S iff

(b) x is a *convex combination* of $x_1, x_2, \ldots, x_n$ iff

(c) A is in *reduced row echelon form* iff

In problems 2 and 3, assume we have a canonical problem $AX = B$ with m independent equations, n variables, and that $k = n - m$.

2(0) Definitions and Theorems.

(a) A set of *basic variables* is a set of variables s.t. ...

3(0) Extreme points have at least k zeros. Characterize extreme points p in terms of (a) the number zeros and (a) in terms of linear independence.

(b) p is extreme iff p is feasible and ...

(c) p is extreme iff p is feasible and ...

4(0) Theorem. If $S = \{v_1, \ldots, v_k\}$ has k vectors in R^n and $A = \langle v_1; \ldots; v_k \rangle$: Answer the following in terms of rank(A), k and n.

S is independent $\Leftrightarrow$

S spans the space $\Leftrightarrow$

S is a basis $\Leftrightarrow$

5(4) For what a does the following system have

$(a+1)x + y = 1, \quad ay = a$

(a) No solutions? (b) Infinitely many solutions?

6(6) $x = (1,2,3)$, $y = (2,3,1)$, $z = (3,1,2)$.

Write $(3,4,5)$ as a linear combination of x, y, z.

7(4) What is the dimension of the space spanned by $(1,2,3)$, $(2,3,4)$ and $(3,4,5)$?

8(3) Give an example of a standard-form linear programming problem with two variables x, y and one inequality which has one maximum and an unbounded set of feasible solutions.

9(8) Sketch the region and find all the maxima.

maximize $z = 2x + y$.

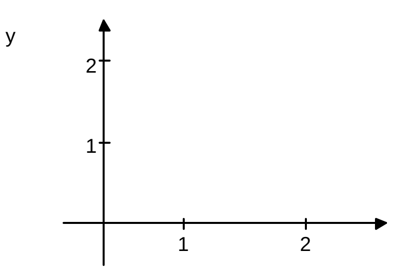
subject to

$2y - x \geq 0$

$y + 2x \leq 2$

$x, y \geq 0$

x	y	z



10(8) Sketch the region and find all the minima.

minimize $w = z - y - 2x$

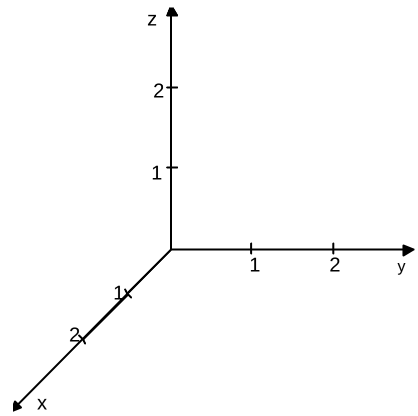
subject to

$2x + y + z = 2$

$x + 2y + z = 2$

$x, y, z \geq 0$

x	y	z	w



11(6) Convert to canonical form.

minimize $z = 2x + y$.

subject to $y + 2x \geq 2, \quad x \geq 0$

12(4) For the canonical problem:

Max $z = x + u + v$

$x - u = 1, \; v = 1, \; x, u, v \geq 0.$

(a) Find a basic solution which is not feasible.

(b) Find a feasible solution which is not basic.

13(7) Translate (but don't solve) into a general linear programming problem:

A new rose dust is being prepared by using two available products: PEST and BUG. Each kilogram of PEST contains 30 grams of carbaryl and 40 grams of Malathion, while each kilogram of BUG contains 40 grams of carbaryl and 20 grams of Malathion. The final blend must contain at least 120 grams of carbaryl and at most 80 grams of Malathion. If each kilogram of PEST costs \$3.00 and each kilogram of BUG costs \$2.50, how many kilograms of each pesticide should be used to minimize the cost?

Let p, b = the number of kilograms of PEST and BUG; c = the cost.

14(7) Translate (but don't solve) into a general linear programming problem:

A coffee packer blends Brazilian coffee and Colombian coffee to prepare two products: Super and Deluxe brands. Each kilogram of Super coffee contains 0.5 kilogram of Brazilian coffee and 0.5 kilogram of Colombian coffee, while each kilogram of Deluxe coffee contains 0.25 kilogram of Brazilian coffee and 0.75 kilogram of Colombian coffee. The packer has 120 kilograms of Brazilian coffee and 160 kilograms of Colombian coffee on hand. If the profit on each kilogram of Super coffee is 20 cents and the profit on each kilogram of Deluxe coffee is 30 cents, how many kilograms of each type of coffee should be blended to maximize profit?

Let s, d be the number kilograms of Super and Deluxe coffee; let p = profit.