

Math 414 Practice Exam 2 /45

Develop enough proficiency to do this in 50 minutes.

You may turn this in for homework credit.

1) Definitions and Theorems.

- (a) p and q are adjacent extremes iff ...
- (b) The maximum number of extremes is ...
- (c) An upward constraint is ...

2) State the Complementary Slackness Theorem.

3(2) (a) Give an example of a basic solution which is not feasible in the following problem.

$$\begin{aligned} \max z &= x+u+v \\ x + u &= 1 \\ v &= 1 \quad x, u, v \geq 0 \\ x, u, v &\geq 0 \end{aligned}$$

(b) For the same problem, give an example of feasible solution which is not basic.

4(4) In the tableau

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- (a) What is the initial set of basic variables?
- (b) The basic solution $x, y, z, u, v, w =$
- (c) What is the entering variable?
- (d) What is the departing variable?

5(7) Restate in canonical form. Solve using the Simplex Algorithm.

Write the two tableaus. Mark the entering & departing variables. List the θ -ratios. Write the final answer - either "Region unbounded with no max," or "max $z=...$ at $x=...$ ". Don't include slack variables in final answer.

$$\begin{aligned} \min z &= -2x+3y \\ \text{with} \\ 2x+y &\leq 6 \\ x+2y &\leq 2 \\ x, y &\geq 0 \end{aligned}$$

6(7) Solve using the Simplex Algorithm.

Write the initial tableau. Pick a set of basic variables, pivot on these and write the resulting tableau. Write the next and last tableau. Mark the entering & departing variables. List the θ -ratios. Write the final answer - either "Region unbounded with no max" or "max $z=...$ at $x=...$ ".

$$\begin{aligned} \max z &= x + 2y \\ \text{with} \\ y + u &= 6 \\ x - 2y - 3v &= 6 \\ x, y &\geq 0, u, v \text{ unrestricted} \end{aligned}$$

7(10) Solve using the two-phase method.

Omit listing θ -ratios. Arrange the slack and artificial variables so that their columns form an identity submatrix of the initial matrix. List the 1st tableau of phase one following the initial pivot. List the last tableau of phase one. List the 1st tableau of phase two following the initial pivot. List the last tableau of phase two.

$$\begin{aligned} \min z &= -2x - y & \text{Answers:} & & \text{Slacks:} \\ \text{with} & & \min z &= & r = \\ r: -2x + y &= 1 & \text{when} & & \\ s: 2x + y &= 5 & x = & & s = \\ & x, y \geq 0 & y = & & \end{aligned}$$

8(3) Give the dual problem.

$$\begin{aligned} \min z &= 8x + 9y \\ \text{with} \\ r: 2x - 3y &= 5 \\ s: 4x + 8y &\leq 6, \\ & x \geq 0, y \text{ unrestricted} \end{aligned}$$

9(6) For the primal problem below, state the canonical problem and the dual problem and sketch the region of feasible solutions. Then calculate the slacks and dual variables.

Primal Problem

$$\begin{aligned} \min z &= -2x - y \\ \text{with} \\ r: -2x + y &= 1 \\ s: 2x + y &= 5 \\ & x, y \geq 0 \end{aligned}$$

10(6)

The primal objective is $\max w = 4x + 2y + 3z$.

The dual objective is $\min w = 12r + 10s + 10t$.

The optimal solution for the primal problem is

$$x = 2, y = 0, z = 4 \text{ with slack } r = 0, s = 1, t = 2.$$

Use the complementary slackness and Duality Theorem to find a solution for the dual problem.

$$\min w = \underline{\hspace{2cm}}, \text{ when } r = \underline{\hspace{2cm}}, s = \underline{\hspace{2cm}}, t = \underline{\hspace{2cm}}.$$