Math 414 Review 1

Exam office hours: Tuesday, 10:00-12:30. Text Wednesday.

Graded homework is placed in the bin on my office door (PSB 318) and may be picked up after 1:00.

Material.

Lectures 1-8. No questions about MatLab or Linsolve. The corresponding text material is on pages 1-101.

Definitions.

Coefficient matrix, augmented matrix, tableau and elementary row operations, tableau and reduced row echelon form, nonsingular, singular, I, $[A|B]$.

Basic and nonbasic variables, basic and general solutions.

Linear combination, independent, subspace spanned by..., dimension, rank(A), convex, closed, bounded, extreme point, closed half-space, convex polyhedron, convex combination, convex hull, convex polytope.

General linear programming problem, feasible solutions, constraints, objective function, standard form, canonical form, slack variable.

Techniques *Be able to*

Use tableau and elementary row operations to transform a matrix to tableau and row reduced element forms..

Use rref to solve systems of equations and compute inverses.

Determine when a system of equations has a unique solution, infinitely many solutions or no solution.

Determine if a set of vectors is a subspace, spans a given space or is independent.

Write a vector as a linear combination of other vectors if possible.

Sketch convex polyhedra and find their extreme points.

Solve linear programming problems. This will be a major part of the test.

Convert general problems to standard and canonical form.

Translate word problems into general linear programming problems.

Suggested Exercises. *All homework exercises plus*

Page Problem

9: 3bc.

20: 1, 3, 5, 7, 9, 13.

28: 5, 7, 9.

32: 3, 5.

41: 1, 3, 5, 11.

81: 1, 3, 5.

91: 1, 3, 5, 7, 12.

57: 1, 3, 5, 7, 9, 11. One or two exam problems will be a variant of one of these. I recommend setting them up before the exam to avoid running out of time during the exam.

Detailed review.

Be able to state the theorems below and answer questions about them. Proofs will not be required.

You will not be asked to prove any theorems. But you will be asked questions about the theorems and their proofs.

THM. If rref($[A|I]$) = $[I|B]$ for some B, then B is the inverse of A, otherwise A is singular and has no inverse.

DEF. rank(A) = the dimension of the subspace spanned by its rows = the dimension ... spanned by its columns.

LEMMA. Elementary row operations on a matrix do not change the subspace spanned by the rows.

LEMMA. A and rref(A) have the same rank.

LEMMA. rank(A) = the number of nonzero rows of rref(A).

THM. If $S = \{v_1, ..., v_k\} \subseteq R^n$ and $A = \langle v_1; ...; v_k\rangle$: S independent $\Rightarrow k \le n$. S spans the space $\Rightarrow n \le k$. S is a basis $\Rightarrow k = n$. S is independent $\Leftrightarrow$ rank(A) $= k$. S spans the space $\Leftrightarrow$ rank(A) $= n$. S is a basis $\Leftrightarrow k =$ rank(A) $= n$.

LEMMA. The line segment $\overline{xy}$ between x and y is the set of points between x and y = all points of the form $ax + by$ where $a + b = 1$, $a, b \ge 0$.

DEF. For any subset S of R^n: S is *convex* iff $x, y \in S \Rightarrow$ all points between x & y are in S. S is *closed* iff it contains all points on its boundary. S is *unbounded* iff it has points arbitrarily far from the origin, S is *bounded* otherwise. An *extreme* point of S is one which is not between two other points of S.

LEMMA. The intersection of convex sets is convex.

DEF. A *closed half-space* is the set of solutions to a non-strict linear inequality. A *convex polyhedron* is an intersection of finitely many closed half-spaces.

A half-space is convex, hence so is a convex polyhedron.

DEF. A *convex combination* of points $x_1, ..., x_n$ is a point of the form $a_1x_1+a_2x_2+...+a_nx_n$ where $a_1+...+a_n=1$ and $a_1 \ge 0, ..., a_n \ge 0$. The *convex hull* of a given set of points is the set of all convex combinations of a given set of points.

A *convex polytope* is the convex hull of a finite set.

LEMMA. The convex hull of a set is the smallest convex set containing the given set.

THM. Every closed bounded convex set is the convex hull of its extreme points. For any closed bounded convex set S, S is a convex polytope iff S has only finitely many extreme points iff S is a convex polyhedron.

THM. For any linear function on a closed bounded convex set, absolute maxima and minima exist and every local maximum or minimum is also an absolute maximum or minimum.

THM. For any linear function f on a closed bounded convex set, the set of maxima is either the extreme point with the largest value among extreme points or the convex hull of all extreme points which have the largest value among extreme points.

Hence finding all maxima reduces to finding the extreme points of largest value.

THM. Every general linear programming problem is equivalent to a programming problem in standard form and to one in canonical form.

THM. In the set S of feasible solutions of a canonical problem, a point is extreme iff it has more 0's than nearby points.

THM. In a canonical problem, X is a basic solution iff $AX = C$ and columns associated with X's positive entries are independent.

LEMMA. For a canonical problem with n variables, m equations and $k = n-m$, every basic solution has $\le m$ positive entries and $\ge k$ zeros.

THM. Given a canonical problem with n variables, m independent equations, $k = n-m$. Every extreme point is a basic solution. Hence every extreme point x has $\ge k$ zeros.

FUNDAMENTAL THEOREM. Extreme points and feasible basic solutions are the same thing.

All exams are closed book: no notes, no text. Bring a calculator and scratch paper. Your scratch paper is not to be turned in; put your answers and all work you wish to show on the exam paper.