Math 414 Review 3

Material.
Lectures 17-24 since Exam 2 except Monday's.

Be able to
Give economic interpretations of slack and dual variables.

Solve economic word problems.
Determine when a problem has a unique optimal solution and if not, find a second optimal solution.
Solve problems with secondary objectives.
Restore feasibility using the dual method.
Solve problems with added constraints.
Determine the range over which a coefficient of the objective function or a constant of some constraint may vary without losing the feasibility or optimality of the solution.

Detailed Review.
Be able to state the theorems below and answer questions about them. As usual, no proofs.

LEMMA. For each feasible solution X for the primal there is a solution W (not necessarily feasible) for the dual which has the same objective value, i.e., $C\bullet X = B\bullet W$.

GIVEN. A canonical tableau with extra variables for a primal problem with constant column B. Let X be the tableau's solution. Let W be the associated solution for the dual problem.
Write the original objective function coefficients (not their negatives) at the top of the tableau.
Label the rows with their basic variables.
To the left of these basic variables, write their objective coefficients (just copy them from the top objective row).
Let C_B be this column of objective coefficients.
Let t_j be the jth column of the tableau or of T (see below).
Let c_j be the objective coefficient at the top.
Let $z_j = C_B\bullet t_j$
Let T be the submatrix of the tableau consisting of the coefficients in the columns of the initial set of basic variables (either slack or extra) which initially formed an identity matrix.

OBJECTIVE ROW THEOREM.
(a) The jth entry in the objective row is $z_j - c_j = C_B\bullet t_j - c_j$.
(b) If the jth column is associated with a variable of the primal problem, then z_j = the left side of the jth dual constraint = $a_{1j}w_1 + a_{2j}w_2 + ... + a_{mj}w_m$ where the constraint is j: $a_{1j}w_1 + a_{2j}w_2 + ... + a_{mj}w_m \geq c_j$.
(c) In this case, $z_j - c_j$, the jth objective row entry, is the slack in the jth dual constraint.
(d) If the jth column of T is associated with the initial basic variable (the extra variable or else the slack variable) for the jth primal constraint, then the value of the jth dual variable w_j = the objective row entry z_j , or $w_j = -z_j$. The sign is determined by the restriction $w_j \geq 0$, or $w_j \leq 0$. If w_j is unrestricted, the Marginal Value Theorem determines the sign. Alternatively, you can just systematically keep track of sign changes (multiplying a constraint by -1 changes the sign of the dual variable).

CONSTANT COLUMN THEOREM. The constant column = $T\cdot B$.

LEMMA. For standard primal problems, the jth dual constraint is satisfied iff $z_j \geq c_j$ iff $z_j - c_j \geq 0$.

PRIMAL-DUAL THEOREM.
(a) For every feasible basic solution of a primal problem, there is an associated solution for the dual problem with the same objective value.
(b) The primal solution is optimal / suboptimal iff the dual solution is optimal / superoptimal.
(c) The primal solution is optimal / suboptimal iff the dual solution is feasible / not feasible.

THEOREM. If the objective coefficient of a final-tableau parameter is 0, select it as an entering variable to get a possibly different optimal basic solution (to be genuinely different some value must change, not just the set of basic variables). If all parameters have nonzero objective row coefficients, the optimal solution is unique.

Suggested Exercises. All homework exercises plus
165: 1, 3, 5, 8, 9, 10.
183: 8, 9, 11.
202: 1, 3, 7, 9, 11.
214: 1, 3, 5, 8.
233: 1, 2, 3.