Due the lecture after Exam 1.

1. Let $T$ be a transition matrix whose diagonal is $[1-p; 1-q]$, with $0 < p, q < 1$.

(a) Find $T$. Hint: What is the row total in a transition matrix?

(b) Find $N$ and a diagonal matrix $D$ such that $T = NDN^{-1}$.

(c) Find a formula for $T^n$.

(d) Find the limit as $n \to \infty$. Write this matrix with no fractions or negative numbers inside the matrix. In the factored answer, $p$ and $q$ both occur three times, the coefficient fraction has 5 symbols, inside the matrix are 4 symbols.

Hint. Factor out common terms to simplify your matrices. E.g. $\begin{pmatrix} \frac{a}{a+b} & \frac{1}{a+b} \\ \frac{1}{a+b} & \frac{b}{a+b} \end{pmatrix} = \frac{1}{a+b} \begin{pmatrix} a & 1 \\ -1 & b \end{pmatrix}$. There should be no fractions inside your matrices. Your first eigenvalue is 1, your first eigenvector is $[1; 1]$. Let the second eigenvalue be $\lambda$. Set the trace($T$) = the sum of the eigenvalues and solve for $\lambda$. Then solve $(A-\lambda I)X = 0$ to get the second eigenvector $X$. The solution is not unique; $X = 0$ is not allowed. These eigenvectors are the two columns of $N$.

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2. An inspector inspects TV sets and classifies them as F(fair), G(good), or E(excellent). Excellent sets are released for sale. Fair and good sets are sent to a shop for adjustment. After adjustment at the shop, a fair set becomes fair (probability 1/3), good (1/3) or excellent (1/3). After adjustment, a good set becomes good or excellent with equal probability. Adjustments are repeated until a set is becomes excellent and is released for sale.

(a) For $n = 1, 2, 3, 4, \ldots$ find the probability it will take exactly $n$ steps to go from a fair state to an excellent state. Use $\sum_{i=0}^{n-1} r^i = (1 - r^n)/(1 - r)$ to simplify to a formula without summations. The answer should have the form $a(r^n - s^n)$.

(b) Find the expected number of steps to go from a fair, good, or excellent state to an excellent state. Simplify as much as you can. Instead of using infinite sums, write equations for say $f(G)$ then solve. The sum of the three answers should be 9/2.