A brick slides down a slope of length $L$. Let $X$ be the distance it slides, i.e., starting from 0, $X$ is the point at which it stops. The probability density function for $X$ is the linear function $f(x) = \frac{2}{L} - \frac{2}{L}x$. It 2/L at 0; 0 at $L$.

(a) Find the probability the brick stops in the middle third of the slope.

(b) The amount of money $E$ you win is the distance the brick slides. $0 if the distance it 0, $L$ dollars if it goes all the way. How much can you expect to win?

A dart is thrown at the unit circle (equation: $x^2 + y^2 = 1$, $y = \sqrt{1-x^2}$ for the upper half, $y = -\sqrt{1-x^2}$ for the lower half). It lands somewhere in the unit circle with a uniform density function $f(x,y)$ (thus all points in the unit circle are equally likely).

(a) Find the marginal distribution function $f_X(x)$ for $x \in [-1,1]$.

(b) Find the conditional density function $f(y|x)$.

A fly lands at a random point (with a uniform distribution) on the interval [0, 1]. Then it hops to the right an exponentially distributed distance with parameter (expected value) 6.

(a) Find the probability that the fly hops into the interval [2, 3]. Give the exact value, no decimals. Recall, the parameter is the reciprocal of the rate $\lambda$.

(b) Given that the the first hop landed at 0 and that the second hop was more than 1, what is the probability that the second hop was more than 3?

Lightning strikes Aloha Tower twice a year on average with a Poisson distribution. What is the probability that Aloha Tower is struck 2 or more times during a one year period? Give exact answers, no decimals.
A bent coin is tossed infinitely many times. $X_n = 1$ if the $n^{th}$ toss is heads, 0 if not. The probability of heads is $p$. Show your work.

(a) Find $E[\sum_{n=1}^{\infty} (X_n/2^n)]$.
(b) Find $E[\lim_{n \to \infty} (X_n)]$.
(c) Find $P[\lim_{n \to \infty} \sum_{n=2}^{\infty} \frac{X_n}{2^{n-1}} = \frac{1}{2}]$. The answer is conditional.

For the Markov chain pictured --

(a) Find the probability that it takes exactly 6 steps to get from $X$ to $W$.

(b) Find the expected number of steps to get from $X$ to $W$.
In a Markov chain experiment, a fly on a $3 \times 3$ grid randomly moves horizontally or vertically to an adjacent square of the grid. From any square, the adjacent squares are equally likely. (For a corner point, there are 2 equally likely landing points; for an interior point, there are 4; for an edge point, 3.) Let the state of the fly be C, E, or M depending on whether or not the fly is on a Corner square, Edge square, or Middle square respectively.

(a)(5) Model this problem as a Markov chain with a $3 \times 3$ transition matrix. Diagonalize the matrix, i.e., write the transition matrix $T$ as $T = NDN^{-1}$. Remember to order the eigenvalues with positives first, then negatives and then 0.

Suppose initially the fly is on a corner square, edge square or the center square with equal probability.

(b)(2) What is the probability distribution after 100 steps?

(c)(2) What is the probability distribution after 101 steps?

(d)(2) As time goes to $\infty$, what proportion of the time, on average, does the fly spend on corner, edge and the center squares respectively?