1. A, B, C, X, Y, Z are states for a Markov chain with the transition probabilities as below.

Let $T^\infty(i,j) = \text{the long-term proportion of time spent in state } j \text{ after starting in state } i$. \(i, j \in \{A, B, C, X, Y, Z\}\).

(a) Find the matrix $T^\infty(i,j)$.

(b) If we start at X, what is the long-term proportion of time spent at A?

(c) If we start at X, what is the probability that we eventually arrive at A?

(d) If we start at A, what is the expected length of time between visits to A?

(e) If we start at A, what is the probability that more than 10% of the time, in the long-run, is spent at A?
A car dealer makes $1000 on the sale of a jeep. He pays $100/month in interest for every jeep on the lot. The number of jeeps per month customers want to buy has the following probability distribution.

<table>
<thead>
<tr>
<th>number D</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>probability</td>
<td>1/10</td>
<td>2/10</td>
<td>4/10</td>
<td>2/10</td>
<td>1/10</td>
</tr>
</tbody>
</table>

If the number of jeeps on the lot at the end of the month is ≤ 1, he increases the number on the lot to 4. Otherwise the number is left unchanged.

- How many jeeps does he have on the lot at the beginning of a month? Give the probability distribution.
- Find the average interest he pays for jeeps per month.
- Find his average monthly sales, in dollars, of jeeps.
3. Male and female patients arrive at a doctor’s office according to independent Poisson processes with respective rates of $\mu$ and $\lambda$ per hour. Given that there were 10 male patients during one 8 hour day, what is the probability that all the female patients arrived in the first 4 hours and all 10 males arrived in the last 4 hours?

4. A machine consists of three parts of types $R$, $S$ and $T$ respectively. All three parts must work in order for the machine to work. For each of the three types of parts, two replacement parts are available of that type. Each part lasts for an exponentially distributed amount of time with rate $\lambda = 3$ before breaking. The three separate parts break independently. Let $X$ be the time the machine ceases to function (for at least one of the three parts, both the part and its two replacements have broken). For any time $t$, find $P[X > t]$. 
At any time, an electric motor is running at one of three speeds: High, Low, Off. Thus the state space is $S = \{H, L, O\}$. It cannot change directly from High to Off or from Off to High. When it is on Low, the next state will be High with $2/3$ and it will be Off with probability $1/3$. The amount of time that the generator stays in each of the three states $H, L, O$ is exponentially distributed with rates $\lambda_H, \lambda_L, \lambda_O$ respectively. Model the speed as a birth-death process and find the long-term probabilities $p_H, p_L, p_O$ of each state.

The preparation of a report requires the successive work of three people: 1, 2, and 3. 1, 2, 3 take an exponentially and independently distributed amount of time to complete their part of the report with rates $\lambda_1, \lambda_2, \lambda_3$ per hour respectively. When 1 has completed his part, he gives to 2. When 2 has completed his part he gives to 3. When 3 has completed his part the report is finished and 1 starts on a new report. What is the long-term expected number of reports that can be finished per hour?