Math 253A - Accelerated Calculus III

Problem sheet 1

01/09/2019

Problem 1

Describe the sets of points (x, y, z) in the three dimensional space that satisfy the following conditions:

- a) $0 \le x \le 1, 0 \le y \le 1.$
- b) $x^2 + y^2 \le 2, z = 3.$
- c) $x^2 + y^2 = 2, y \ge 0.$
- d) The set of points equidistant from the origin and the point (0, 0, 2). Give an equation for this set.

Problem 2

a) Find the radius and the center of the sphere

$$3x^2 + 3y^2 + 3z^2 + 2y - 2z + 1 = 10.$$

- b) Find the equation of the sphere with center (2, 6, 4) that passes through the origin.
- c) Give an inequality to describe the half space of points on and below the xy-plane.
- d) Find a pair of equations for the intersection of the cone $z = x^2 + y^2$ with the plane z = 4. Describe the set of all intersection points.

Problem 3

- a) Compute the distance between the points P = (1, 2, 3) and Q = (3, 2, 1).
- b) Give the component form of the vectors \overrightarrow{PQ} and \overrightarrow{QP} . Show that $\overrightarrow{PQ} = -\overrightarrow{QP}$.
- c) Calculate the magnitudes of the vectors \overrightarrow{PQ} and $-4\overrightarrow{QP} + \overrightarrow{PQ}$.
- d) Find the components of the vector $3\vec{u} 2\vec{v}$ for $\vec{u} = \langle -1, 0, -3 \rangle$ and $\vec{v} = \langle 1, -1, 2 \rangle$.