

Math 253A - Accelerated Calculus III

Homework sheet 3

Due 02/02/2018

To read: 11.6, 12.1 and 12.2 in the book.

Problem 1

a) Sketch the following two surfaces in the three dimensional space:

$$(i) y^2 + z^2 = 4z^2, \quad (ii) (x - 2)^2 + (y - 1)^2 = 1.$$

b) Give a parametrization of the part of the circle $x^2 + y^2 = 4$ in the first quadrant of the xy -plane (i.e. $x \geq 0$ and $y \geq 0$). Use $(2, 0)$ as initial and $(0, 2)$ as end point.

c) Give a parametrization of the tangent line at the circle curve in b) at the point $(x_0, y_0) = (\sqrt{2}, \sqrt{2})$.

Problem 2

We consider a particle moving in time t along the curve given by

$$\vec{r}(t) = 2 \cos t \vec{i} + 3 \sin t \vec{j} + 2t \vec{k}, \quad t = [0, 2\pi].$$

a) Graph this curve. Is it a closed curve?

b) Give the velocity and the acceleration of the particle for any $t \in [0, 2\pi]$.

c) Find the particle's speed and direction of motion at the time $t_0 = \pi/2$.

d) Give a parametrization of the tangent line to the curve \vec{r} at $t_0 = \pi/2$.

Problem 3 (Throwing a baseball)

A baseball is thrown from the stands 32 ft above the field at an angle of 30° up from the horizontal (Have a look at Figure 12.10 and the corresponding formula (7) in the book). When and how far away will the ball strike the ground if its initial speed is $v_0 = 32$ ft/sec?

Problem 4

a) Evaluate the following vector valued integral:

$$\int_1^2 \left((6 - 6t) \vec{i} + 3\sqrt{t} \vec{j} + \frac{4}{t^2} \vec{k} \right) dt.$$

b) Solve the following initial value problem for \vec{r} as a vector valued function of t :

$$\vec{r}''(t) = -\vec{i} - \vec{j} - \vec{k},$$

with the initial conditions $\vec{r}(0) = \langle 10, 10, 10 \rangle$ and $\vec{r}'(0) = \langle 0, 0, 0 \rangle$.