

Math 253A - Accelerated Calculus III

Homework sheet 5

Due 02/19/2018

To read: Section 13.1, 13.2 in the book.

Problem 1

For the following bivariate functions: (i) find the domain of the function, (ii) find the range of the function, (iii) describe the level sets of the function, (iv) determine whether the domain is an open or a closed region, (v) determine whether the domain is a bounded or an unbounded set.

- $f(x, y) = x^2 - y^2$,
- $f(x, y) = \sqrt{y - x}$,
- $f(x, y) = y/x^2$,
- $f(x, y) = \frac{1}{\sqrt{16 - x^2 - y^2}}$.

Problem 2

- Sketch the surface $z = f(x, y)$ for the function $f(x, y) = 4 - y^2$.
- Sketch the level set of the function $f(x, y, z) = y^2 + z^2$ that contains the point $(1, 1, 0)$.
- Find an equation for the level set of the function $f(x, y) = \sqrt{x^2 - 1}$ that contains the point $(1, 0)$.
- Find an equation for the level set of the function $f(x, y, z) = \ln(x^2 + y + z^2)$ through the point $(-1, 2, 1)$.

Problem 3

- Let Δ be the closed triangle determined by the vertices $A = (0, 0)$, $B = (2, 2)$ and $C = (4, 0)$. Sketch Δ and give a mathematical description of the set Δ , the boundary $\partial\Delta$ and the open interior $\overset{\circ}{\Delta}$.
- Give the domain $D \subset \mathbb{R}^3$ of the trivariate function $f(x, y, z) = \frac{1}{\sqrt{1 - x^2 - y^2 - z^2}}$. Is D an open or a closed domain? What is the range of f ?

Problem 4

Compute the following limits or explain why they don't exist.

- $\lim_{(x,y,z) \rightarrow (0,0,2)} \sqrt{1 + x^2 + y^2 + z^2}$,
- $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2}{x^2 + y^2}$,
- $\lim_{(x,y) \rightarrow (1,0)} \frac{(x-1)y}{(x-1)^2 + y^2}$,
- $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y^2}{x^2 + y^2}$.