University of Hawai'i at Mānoa | Department of Mathematics
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# Math 253A - Accelerated Calculus III

**Homework sheet 6** **Due 02/27/2018**

**To read:** Section 13.3, 13.4 and 13.5 in the book.

**Problem 1**
Each of the following two exercises gives a function $f$ and a positive number $\epsilon$. Show that you can find a proper $\delta > 0$:

a) Given $f(x,y) = \frac{y}{x^2+1}$, $\epsilon = 0.05$, find a $\delta > 0$ such that for all $(x,y)$ with $\sqrt{x^2+y^2} < \delta$ you get $|f(x,y) - f(0,0)| < \epsilon$.

b) Given $f(x,y,z) = xyz$, $\epsilon = 0.008$, find a number $\delta > 0$ such that for all $(x,y)$ with $\sqrt{x^2+y^2+z^2} < \delta$ you get $|f(x,y,z) - f(0,0,0)| < \epsilon$.

**Problem 2**
Find the partial derivatives $\partial f/\partial x$ and $\partial f/\partial y$ (and $\partial f/\partial z$ if $f$ depends also on $z$) for the following functions:

a) $f(x,y) = e^{xy} \ln y$.

b) $f(x,y) = \cos^2(2x - y^2)$.

c) $f(x,y,z) = x - \sqrt{y^2+z^2}$.

d) $f(x,y,z) = 1/\sqrt{x^2+y^2+z^2}$.

**Problem 3**
Consider the function

$$f(x,y) = \begin{cases} \frac{x^2y^2}{x^4+y^4} & (x,y) \neq (0,0), \\ 0 & (x,y) = (0,0) \end{cases}.$$

a) Calculate the first partial derivatives $f_x$ and $f_y$ of $f$ at $(0,0)$ (use the limit definition for this).

b) Calculate the first partial derivatives of $f$ at all other points $(x,y) \neq (0,0)$.

c) Is the function $f$ continuous at the origin?

d) Is the function differentiable at the origin?

**Problem 4**

a) Find the direction $\vec{v}$ in which the function $f(x,y,z) = \sqrt{x^2+xy+z^2}$ attains its maximal directional derivative at the point $(x_0,y_0,z_0) = (1,4,2)$. Make sure your answer is a unit vector.

b) What is the maximal directional derivative of $f(x,y) = \sqrt{xy}$ at the point $(\sqrt{3},1)$?