Math 253A - Accelerated Calculus III

Homework sheet 10 **Due 04/06/2018**

To read: Section 14.5, 14.6, 14.7 in the book.

Problem 1 (§14.4 #24)
In polar coordinates, the average value of a function over a region R is given by

$$\frac{1}{\text{Area}(R)}\iint_R f(r,\theta)r\,dr\,d\theta.$$

Compute the average height of the cone $z = \sqrt{x^2 + y^2}$ above the disk $R = \{(x,y) : x^2 + y^2 \le a^2\}$ with radius a in the xy-plane.

Problem 2
Sketch the solid region of integration for

$$\int_0^1 \left(\int_y^1 \left(\int_0^z F(x,y,z)dx \right) dz \right) dy.$$

How many faces, corners and edges does it have? Write the iterated integral as five other iterated integrals.

Problem 3
Find the mass and center of mass of the solid D with density function $\delta(x,y,z) = y$ where D is the tetrahedron bounded by the planes $x = 0$, $y = 0$, $z = 0$ and $x + y + z = 1$.

Problem 4
Use cylindrical coordinates to evaluate the integral:

$$\iiint_D ze^{x^2+y^2+z^2}\,dx\,dy\,dz$$

where D is the portion of the unit ball $x^2 + y^2 + z^2 \le 1$ that lies in the first octant. (The first octant is the region where $x \ge 0$, $y \ge 0$ and $z \ge 0$)