

Math 253A - Accelerated Calculus III

Homework sheet 11

Due 04/11/2018

To read: Section 14.7, 14.8 in the book.

Problem 1

Find the mass and the center of mass of a brick occupying the region

$$D = \{(x, y, z) : 0 \leq x \leq 1, 0 \leq y \leq 2, 0 \leq z \leq 1\}$$

with density function $\delta(x, y, z) = 2 + xy - 2z$. (Check Table 14.1 in the book for definitions)

Problem 2

Rewrite the integral

$$\int_{-2}^2 \left(\int_0^{\sqrt{4-y^2}} \left(\int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} y^2 \sqrt{x^2 + y^2 + z^2} dz \right) dx \right) dy$$

using spherical coordinates and evaluate it.

Problem 3

Evaluate the double integral

$$\iint_R \frac{x-y}{x+y} dx dy$$

where R is the square with vertices $(0, 2)$, $(1, 1)$, $(2, 2)$ and $(1, 3)$.

Hint: an appropriate transformation to new coordinates s and t might simplify the calculations.

Problem 4

Use the transformation $x = s^2$, $y = t^2$ and $z = u^2$ to compute the volume of the region bounded by the surface $\sqrt{x} + \sqrt{y} + \sqrt{z} = 1$ and the planes $x = 0$, $y = 0$ and $z = 0$. What does the region look like in the stu space? Can you draw the original solid in the xyz space?