

## Math 253A - Accelerated Calculus III

Homework sheet 12

Due 04/18/2018

**To read:** Section 15.1, 15.2 in the book.

### Problem 1

Let  $C$  be the boundary of the rectangle with vertices  $(0, 0)$ ,  $(1, 0)$ ,  $(1, 3)$  and  $(0, 3)$ . The curve  $C$  consists of the four line segments

- $C_1$  is the segment from  $(0, 0)$  to  $(1, 0)$ ,
- $C_2$  is the segment from  $(1, 0)$  to  $(1, 3)$ ,
- $C_3$  is the segment from  $(1, 3)$  to  $(0, 3)$ ,
- $C_4$  is the segment from  $(0, 3)$  back to  $(0, 0)$ .

Draw the curve  $C$  and parametrize the line segments  $C_1$ ,  $C_2$ ,  $C_3$  and  $C_4$ . Compute the line integral of the first kind

$$\int_C (x + y + z) ds.$$

### Problem 2

Find the mass and center of mass of a wire lying along a quartered circle  $x^2 + y^2 = 9$  in the first quadrant of the  $xy$ -plane (i.e.  $x \geq 0$ ,  $y \geq 0$ ,  $z = 0$ ) and having density  $\delta(x, y, z) = 5xy$ . (For the calculation, find first a suitable parametrization of the given curve)

### Problem 3

Find the work done by the force field

$$\mathbf{F}(x, y, z) = \langle x - y^2, y - z^2, z - x^2 \rangle$$

on a particle moving along the line segment from  $(0, 0, 1)$  to  $(2, 1, 0)$ .

### Problem 4

Determine the work integral  $\int_C \mathbf{F} \cdot d\mathbf{r}$  where the force field  $\mathbf{F}$  is given by  $\mathbf{F}(x, y, z) = 3x\mathbf{i} + 3y\mathbf{j} + 3z\mathbf{k}$  and the parametrization of the curve  $C$  is given by  $\mathbf{r}(t) = \langle t^2 + 1, t^2 - 1, t^2 - 2t \rangle$  with  $0 \leq t \leq 2$ .