University of Hawai'i at Mānoa — Department of Mathematics
W. Erb — Spring 2018

Math 253A - Accelerated Calculus III

Homework sheet 13 **Due 04/25/2018**

To read: Section 15.3, 15.4 in the book.

Problem 1
Let C be the boundary of the triangle with vertices $(1,1)$, $(2,0)$ and $(1,3)$. The closed curve C consists of the three line segments

- C_1 is the segment from $(1,1)$ to $(2,0)$,
- C_2 is the segment from $(2,0)$ to $(1,3)$,
- C_3 is the segment from $(1,3)$ to $(1,1)$.

Draw the curve C and parametrize the line segments C_1, C_2 and C_3.

Compute the flux of the velocity field $\vec{\mathbf{F}}(x,y,z) = (x+y)\vec{\mathbf{i}} + (x^2+y^2)\vec{\mathbf{j}}$ outward across the given triangle C.

Problem 2

a) Use the component test to show that $\vec{\mathbf{F}}(x,y,z) = \frac{2x}{y}\vec{\mathbf{i}} + \frac{1-x^2}{y^2}\vec{\mathbf{j}} + \vec{\mathbf{k}}$ is a conservative vector field in the region $D = \{(x,y,z) : \; y > 0\}$.

b) Find a potential function f for $\vec{\mathbf{F}}$.

c) Determine the value of the work integral $\int_C \vec{\mathbf{F}}(\vec{\mathbf{r}}) \cdot \vec{\mathbf{dr}}$ along the line segment from $(0,1,0)$ to $(1,2,4)$.

Problem 3 Use Green's Theorem to evaluate the following line integrals.

(a) Find $\oint_C \vec{\mathbf{F}}(\vec{\mathbf{r}}) \cdot \vec{\mathbf{dr}}$ where $\vec{\mathbf{F}}(x,y) = \sqrt{x^2+1}\;\vec{\mathbf{i}} + \tan^{-1} x\;\vec{\mathbf{j}}$ and C is the triangle from $(0,0)$ to $(1,1)$ to $(0,1)$ to $(0,0)$.

(b) Find $\int_C \vec{\mathbf{F}}(\vec{\mathbf{r}}) \cdot \vec{\mathbf{dr}}$ where $\vec{\mathbf{F}}(x,y) = \langle e^{-x}+y^2, e^{-y}+x^2 \rangle$ and C is the arc of the curve $y = \cos x$ from $(-\pi/2, 0)$ to $(\pi/2, 0)$. (Note that this isn't a closed curve!)

Problem 4 Find the area of the region enclosed by the parameterized curve $\vec{\mathbf{r}}(t) = \langle t-t^2, t-t^3 \rangle$, $0 \le t \le 1$ (Green's theorem might be a useful tool for this). Try to graph the curve.