University of Hawai'i at Mānoa | Department of Mathematics
W. Erb | Spring 2018

Math 253A - Accelerated Calculus III

Homework sheet 14 **Due 05/02/2018**

To read: Section 15.5, 15.6 and 15.7 in the book.

Problem 1
Consider the spherical band S which is the portion of the sphere $x^2 + y^2 + z^2 = 4$ between the planes $z = -1$ and $z = \sqrt{3}$. Parameterize the surface S and compute the surface area of the spherical band S using a double integral.

Problem 2 Let the surface S be parametrized by $\vec{\mathbf{r}}(s,t) = \langle s, s+t, t\rangle$, $0 \le s \le 1$, $0 \le t \le 2$. Find

$$\iint_S (x^2 + y^2 + z^2)\, d\sigma.$$

Problem 3 Evaluate the surface integral

$$\iint_S \vec{\mathbf{F}} \cdot \vec{\mathbf{n}}\, d\sigma,$$

where $\vec{\mathbf{F}}(x,y,z) = \langle x, y, 2z\rangle$, the surface S is the part of the paraboloid $z = 4 - x^2 - y^2$ that lies above the unit square $0 \le x \le 1,\ 0 \le y \le 1$ and $\vec{\mathbf{n}}$ is the upward pointing unit normal vector to the surface S.

Problem 4 Use Stokes' Theorem to calculate

$$\iint_S \operatorname{curl} \vec{\mathbf{F}} \cdot \vec{\mathbf{n}}\, d\sigma,$$

where $\vec{\mathbf{F}}(x,y,z) = \langle -y, x, xyz\rangle$ and S is the part of the sphere $x^2 + y^2 + z^2 = 25$ that lies below the plane $z = 4$, oriented so that the unit normal vector at the south pole $(0,0,-5)$ is $\langle 0,0,-1\rangle$.