MATH 442 – HW 3

NAME:

You will not be able to do everything in these problems using only what was covered in class on Tuesday 1/28. By the end of class on Thursday 1/30 we will have covered everything you need.

(1) Answer the following questions about the function \( f : \mathbb{R}^2 \to \mathbb{R} \) defined by

\[
f(x, y) = \begin{cases} 
\frac{x^3}{x^2+y^2}, & \text{if } (x, y) \neq (0, 0) \\
0, & \text{if } (x, y) = (0, 0)
\end{cases}
\]

a) For which \( u \in \mathbb{R}^2 \) does the directional derivative \( f'(0, 0; u) \) exist?

b) Do the partial derivatives \( D_1f(0, 0) \) and \( D_2f(0, 0) \) exist?

c) Is \( f \) differentiable at \((0, 0)\)?

d) Is \( f \) continuous at \((0, 0)\)?

This is exercise 4, p.49 of Munkres.

(2) Same questions for the function \( g : \mathbb{R}^2 \to \mathbb{R} \) defined by

\[
g(x, y) = \begin{cases} 
\frac{xy}{x^2+y^2}, & \text{if } (x, y) \neq (0, 0) \\
0, & \text{if } (x, y) = (0, 0)
\end{cases}
\]

This is exercise 2, p.48 of Munkres.

(3) Same questions for the function \( h : \mathbb{R}^2 \to \mathbb{R} \) defined by

\[
h(x, y) = \begin{cases} 
\sqrt{|xy|}, & \text{if } (x, y) \neq (0, 0) \\
0, & \text{if } (x, y) = (0, 0)
\end{cases}
\]

This is exercise 6, p.49 of Munkres.