Evaluation Formulas and Theorems

All curves and surfaces are assumed to be (piecewise) smooth; all vector fields are assumed to be smooth.

**Line Integrals**

*Setup:* $C$ an oriented curve, $\vec{F}$ a vector field

*Evaluation formula:* \[ \int_C \vec{F} \cdot \vec{T} \, ds = \int_{t=a}^{t=b} \vec{F}(t) \cdot \vec{v}(t) \, dt, \]

where $\vec{v}(t)$ is the velocity of a smooth parametrization $\vec{r}(t)$ of $C$ with domain $a \leq t \leq b$.

**Surface Integrals**

*Setup:* $S$ an oriented surface, $\vec{F}$ a vector field

*Evaluation formula:* \[ \iint_S \vec{F} \cdot \vec{n} \, dS = \iint_D \vec{F}(u,v) \cdot (r_u(u,v) \times r_v(u,v)) \, dA(u,v), \]

where $\vec{r}(u,v)$ is an oriented smooth parametrization of $S$ with domain $D$, that is, $\vec{r}(u,v)$ is a smooth parametrization of $S$ for which the vector $r_u(u,v) \times r_v(u,v)$ has the same direction as the given orientation vector $\vec{n}$.

**Green’s Theorem**

*Setup:* $C$ a simple closed curve, the boundary of a region $D$ in the plane, $C$ oriented counterclockwise, $\vec{F} = \langle P, Q \rangle$ a vector field

*Green’s Theorem (primitive form):* \[ \oint_C P \, dx + Q \, dy = \iint_D \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) \, dA \]

*Green’s Theorem (flow form):* \[ \int_C \vec{F} \cdot \vec{T} \, ds = \iint_D \left( \nabla \times \vec{F} \right) \cdot \vec{k} \, dA \]

*Green’s Theorem (flux form):* \[ \int_C \vec{F} \cdot \vec{n} \, ds = \iint_D \nabla \cdot \vec{F} \, dA, \]

where $\vec{n}$ is the unit normal vector on $C$ pointing out of $D$.

**Stokes’ Theorem**

*Setup:* $S$ a surface with boundary curve $C$, $S$ and $C$ are compatibly oriented, $\vec{F}$ a vector field

*Stokes’ Theorem:* \[ \iint_S \left( \nabla \times \vec{F} \right) \cdot \vec{n} \, dS = \oint_C \vec{F} \cdot \vec{T} \, ds \]

**Divergence Theorem**

*Setup:* $E$ a solid region in space with boundary surface $S$, $S$ oriented outward from $E$, $\vec{F}$ a vector field

*Divergence Theorem:* \[ \iiint_E \nabla \cdot \vec{F} \, dV = \iint_S \vec{F} \cdot \vec{n} \, dS \]

*Note:* The theorems, suitably interpreted, remains true for more general regions and curves.