MATH 321 - PRACTICE MIDTERM

Logic: Explain the difference between a statement and a predicate. Let \( A \) and \( B \) be statements. Using truth tables verify that:
\[
(A \implies B) \iff -(A \land -B) \iff (-A \lor B).
\]

Induction: Prove by induction that
\[
\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}.
\]

Partial Orders: Let \( S \) be a set.

a) Define what we mean by a partial order on \( S \).

b) Define what we mean by a total order on \( S \).

Define an order relation on \( \mathbb{N} \) by \( a \leq b \) if and only if \( a \) divides \( b \): \( a \leq b \iff a|b \).

c) Show that \( \leq \) is a partial order on \( \mathbb{N} \).

d) Show that \( \leq \) is not a total order on \( \mathbb{N} \).

e) Show that \( \leq \) is a total order on the set \( \{ p^n : n \in \mathbb{N} \} \)

f) Does \( \mathbb{N} \) have any minimal elements for \( \leq \)?

g) Does the set \( \mathbb{N} - \{ 1 \} \) have minimal elements for \( \leq \)?

h) Does the set \( \{ p^n : n \in \mathbb{N} \} \) have minimal elements for \( \leq \)?

Equivalence Relations: Let \( S \) be a set.

a) Define what we mean by equivalence relation on \( S \).

b) Let \( \sim \) be an equivalence relation on \( S \). Define what we mean by the equivalence class of \( a \in S \).

c) For \( a \in S \) let \([a]\) denote the equivalence class of \( a \) (this was denoted \( T_a \) in the textbook). Prove:
\[
[a] = [b] \iff [a] \cap [b] \neq \emptyset \iff a \sim b
\]

Graphs: Let \( G = (V, E) \) be a graph.

a) Define what we mean by a walk in \( G \).

b) Define what we mean by a cycle in \( G \).

c) Define what we mean when we say \( G \) is connected.

d) Show that if \( G \) is connected, and the edge \( e \) is contained in a cycle, then the graph \( G' = (V, E - \{ e \}) \) is connected.