2000g:53101 53D50 22E15 53C26

Brylinski, Ranee (1-PAS)

Geometric quantization of real minimal nilpotent orbits. (English. English summary)

Symplectic geometry.

Differential Geom. Appl. **9** (1998), *no.* 1-2, 5–58.

The author studies the geometric quantization of nilpotent orbits $\mathcal{O}_{\mathbf{R}}$ of a real semisimple Lie group $G_{\mathbf{R}}$ by using a Kahler polarization invariant under a maximal compact subgroup $K_{\mathbf{R}}$. This Kahler structure is induced from a hyper-Kahler structure on the corresponding orbit of the complex semisimple group $G_{\mathbf{C}}$, discovered by P. Kronheimer, and was investigated by the author in a previous paper [in Representation theories and algebraic geometry (Montreal, PQ, 1997), 85–125, Kluwer Acad. Publ., Dordrecht, 1998; MR 2000a:53151]. Using work of Vergne the author realizes the Lie algebra $g_{\mathbf{R}}$ of $G_{\mathbf{R}}$ as a Lie algebra of rational functions on T^*Y, where $Y = (\mathcal{O}_{\mathbf{R}}, J)$.

The quantization problem can then be transformed into a quantization problem on T^*Y, which is solved for strongly minimal nilpotent orbits in the non-Hermitian case. The Hilbert space consists of holomorphic half-forms on Y. The Lie algebra acts by explicit pseudodifferential operators on half-forms, where the "energy operator", which is the quantization of the Kahler potential has been inverted. The Lie algebra representation exponentiates to a minimal ladder representation of a cover of $G_{\mathbf{R}}$. The paper completes the work begun by the author with B. Kostant [in Functional analysis on the eve of the 21st century, Vol. 1 (New Brunswick, NJ, 1993), 13–63, Progr. Math., 131, Birkhauser Boston, Boston, MA, 1995; MR 96m:22025] which dealt with the three cases where $G_{\mathbf{R}}$ is a split group of type E_6, E_7, E_8.

{For the entire collection see MR 99b:58007.}

Christopher T. Woodward (1-RTG)

2000f:53113 53D20

Marsden, Jerrold E. (1-CAIT-CN)**; Misiolek, Gerard** (1-NDM)**;**
Perlmutter, Matthew (1-CA)**; Ratiu, Tudor S.** (1-UCSC)

Symplectic reduction for semidirect products and central extensions. (English. English summary)

Symplectic geometry.

Differential Geom. Appl. **9** (1998), *no.* 1-2, 173–212.

Let M be a Lie group that acts on a symplectic manifold (P, Ω) in Hamiltonian fashion. Let N be a normal subgroup. The main result is that under suitable technical hypotheses the symplectic quotient of P by M can be constructed in stages, that is, by first reducing P by N, and then (roughly speaking) reducing by the quotient group M/N. For full details the reader is referred to a companion paper by the same authors ["Symplectic reduction by stages", in preparation]. In the case that M is compact and connected, the result is due to R. Sjamaar and E. Lerman [Ann. of Math. (2) 134 (1991), no. 2, 375–422; MR 92g:58036 (Theorem 4.4)]. The authors give many interesting examples of Hamiltonian actions of non-compact group extensions to which their theory applies.

{For the entire collection see MR 99b:58007.}

Christopher T. Woodward (1-RTG)

2000a:53136 53Dxx 57R17

Eliashberg, Yasha [Eliashberg, Yakov M.] (1-STF)

Symplectic topology in the nineties. (English. English summary)

Symplectic geometry.

Differential Geom. Appl. **9** (1998), *no.* 1-2, 59–88.

This survey paper begins with a concise, well-written introduction to the basic notions of symplectic and contact topology. The main sections of the paper give snapshots of the progress in a few specific areas, giving just enough background to motivate and state a few theorems of interest in each area. The survey was intended to convey some of the progress most of interest to the author, and does not attempt to give a complete review of current developments. However, the extensive bibliography, broken into 14 sections and containing 237 references, provides an excellent window onto a broad range of topics in symplectic topology.

The main sections of the paper concern contact 3-manifolds, the Hofer geometry on the group of Hamiltonian symplectomorphisms, Donaldson's theory of approximately complex codimension-2 submanifolds, applications of Taubes' work on Seiberg-Witten theory to symplectic 4-manifolds and brief mention of a few applications of pseudo-holomorphic curves to symplectic 4-manifolds, and finally generating functions and their applications to Lagrangian and Legendrian submanifolds. At the end, a set of interesting open questions is provided, followed by quick mention of some of the other areas of advancement in symplectic topology.

The sections of the bibliography are titled: Books and surveys; Before the birth of symplectic topology; Emergence of symplectic topology; Contact 3-manifolds; Holomorphic curves in contact geometry; Floer homology theory; Hofer geometry; Holomorphic curves in symplectic 4-manifolds; Approximately complex submanifolds, Lefschetz fibrations, etc.; Quantum cohomology and mirror symmetry; Symplectic topology and Hamiltonian dynamics, symplectic invariants; Topology of Lagrangian embeddings; Generating functions approach; and Other references.

Here are two clarifications that might be helpful for the reader not familiar with symplectic manifolds. The first statement on page 72, concerning the existence of complex structures on symplectic submanifolds, refers to a local phenomenon as there exist symplectic manifolds which do not admit any complex structure. The other clarification concerns Theorem 5.3: the embedded 2-sphere is in fact a symplectic sphere.

{For the entire collection see MR 99b:58007.}

Margaret F. Symington (1-IL)

99i:58066 58F05 53C05 58H05

Weinstein, Alan (1-CA)

Poisson geometry. (English. English summary)

Symplectic geometry.

Differential Geom. Appl. **9** (1998), *no.* 1-2, 213–238.

From the introduction: "Poisson geometry has become an active field of research during the past 30 years or so, stimulated by connections with a number of areas, including harmonic analysis on Lie groups, infinite-dimensional Lie algebras, mechanics of particles and continua, singularity theory, and completely integrable systems, just to mention a few examples."

This paper is a survey of Poisson geometry, made by one of the principal contributors to this area, but here he places special attention on his own interests, such as concretely global questions, completeness, Lie algebroids, Lie groupoids and Poisson Lie groups. Each part of the survey is composed of basic definitions and facts, as well as the principal results and a lot of references where these results can be found. The reading is also accompanied by motivating comments, and finally open problems are suggested. The paper finishes with a large number of references (144) which, although not complete, are very important. This paper is a little sketch of the book in preparation by A. Cannas da Silva and the author [Geometric models for noncommutative algebras, Center Pure Appl. Math., Dept. Math., Univ. California Berkeley, Berkeley, CA, to appear] (available at http://math.berkeley.edu/~alanw/).

This survey begins with the introduction, where the objectives are pointed out. Section 2 is devoted to Poisson maps and their relation with co-isotropic submanifolds. The local structure of Poisson manifolds is described in Section 3, including totally degenerate structures, the linearization problem and quadratic Poisson structures. In Section 4 a few remarks on the classification of regular Poisson structures are made. Poisson cohomology and homology are explained in Section 5. The different notions of completeness in Poisson geometry are considered in Section 6, i.e., complete functions, complete manifolds, complete maps, completeness of Poisson Lie groups and completeness in Poisson cohomology classes. The concepts of Lie algebroids and Lie groupoids are described in Section 7 and then, in Section 8, the study of symplectic groupoids, groupoid actions, the relation of Poisson Lie groups with the groupoids, Poisson group actions, Poisson homogeneous spaces and moment/momentum maps, is developed. Section 9 is devoted to modular theory and Section 10 to self-similarities and Liouville vector fields. Some generalizations of Poisson structures are mentioned in Section 11, such as Jacobi, bi-Hamiltonian, Poisson-Nijenhuis, Dirac or Nambu structures. The survey finishes with some miscellaneous examples and questions about Poisson geometry.

{For the entire collection see MR 99b:58007.}

Raúl Ibáñez (E-BILBS)

99e:58085 58F06 58H15

Karasev, Mikhail (RS-MIEM)

Advances in quantization: quantum tensors, explicit star-products, and restriction to irreducible leaves. (English. English summary)

Symplectic geometry.

Differential Geom. Appl. **9** (1998), *no.* 1-2, 89–134.

This paper deals with a star product $\star$ on a Poisson manifold M. In the first part, several concepts related to quantized calculus are introduced, e.g., the lunar product, quantum vector fields, quantum tensors, the quantum Lichnerowicz-Schouten bracket and the quantum de Rham complex. In particular, by using the quantum $\star$-tensor K, an explicit expression of the product $\star$ is obtained, and also the condition is described by means of K that the given star product has representations by h-pseudodifferential operators. Here the quantum $\star$-tensor K is given for a local coordinate system $(\xi^1, \cdots, \xi^m)$ as $K = (K^{ls})$, with

$$K^{ls} = \frac{i}{h}(\xi^l \star \xi^s - \xi^s \star \xi^l), \quad l, s = 1, 2, \cdots, m.$$

In the second part, the main subject is star products on Poisson manifolds M with partial complex structure. Here the partial complex strucure is defined by an atlas with local coordinates $(A^1, \cdots, A^k, C^1, \cdots, C^d)$, where $A = (A^1, \cdots, A^k)$ is real, $C = (C^1, \cdots, C^d)$ is complex and local coordinate transformations are holomorphic with respect to the C-part. Due to the partial complex structure, a concept similar to that of Wick star product is considered which is called a normal product. Hence, the author describes the construction of the normal product from the Weyl $\star$-product on M and the representation of the normal product algebra as a differential operator algebra on a certain Hilbert space: Denote by $\star_a$ the corresponding product of symbols of differential operators; then the homomorphism is constructed from the normal $\star$-product algebra onto the $\star_a$-product algebra, and also an explicit construction of the $\star_a$-product algebra as an algebra of differential operators on the Hilbert space of certain antiholomorphic functions is given. Moreover, an explicit formula is obtained for the expansion of the product $\star_a$ whose terms consist of the differential invariants of (pseudo-) Kahler form.

The concepts of quantum normal coordinates and quantum irreducible leaves are introduced. The quantum restriction is defined and the $\star_a$-product can be restricted to the quantum irreducible leaves.

{For the entire collection see MR 99b:58007.}

Akira Yoshioka (J-SUTS)

99e:58072 58F05 14D20 14L30 58F06

Kirwan, Frances (4-OX)

Momentum maps and reduction in algebraic geometry. (English. English summary)

Symplectic geometry.

Differential Geom. Appl. **9** (1998), *no.* 1-2, 135–171.

This paper surveys work of the last fifteen years on the geometry of the momentum map and its applications to symplectic reduction. The main theme is to describe how the symplectic reduction at a coadjoint orbit varies when the orbit varies. This gives an approach to the geometry and topology of moduli spaces, via the connections between symplectic reduction and Mumford's geometric invariant theory. After an introduction to these topics, the author gives a nice overview of recent developments concerning symplectic cutting, singular reductions, the cohomology ring of symplectic reductions, and the Guillemin-Sternberg conjecture that quantization commutes with reduction. Finally, she shows how these methods can be adapted to determine the intersection pairing in the cohomology ring of moduli spaces of vector bundles on algebraic curves [see L. C. Jeffrey and F. C. Kirwan, Ann. of Math. (2) 148 (1998), no. 1, 109–196; MR 2000c:14045].

{For the entire collection see MR 99b:58007.}

Michel Brion (F-GREN-F)

99b:58007 58-06

★**Symplectic geometry.**

Edited by Mark J. Gotay.

Differential Geom. Appl. **9** (1998), no. 1-2.

North-Holland Publishing Co., *Amsterdam*, 1998. *pp. i–ii and* 1–238.

Contents: Ranee Brylinski, Geometric quantization of real minimal nilpotent orbits (5–58); Yasha Eliashberg [Yakov M. Eliashberg], Symplectic topology in the nineties (59–88); Mikhail Karasev, Advances in quantization: quantum tensors, explicit star-products, and restriction to irreducible leaves (89–134); Frances Kirwan, Momentum maps and reduction in algebraic geometry (135–171); Jerrold E. Marsden, Gerard Misiolek, Matthew Perlmutter and Tudor S. Ratiu, Symplectic reduction for semidirect products and central extensions (173–212); Alan Weinstein, Poisson geometry (213–238).

{The papers are being reviewed individually.}