Math 331 Assignment #3 due Friday Feb. 22/08

1. p. 55 # 18 – 20.

2. p. 57 # 34.

3. Let $S_k = 1 + \frac{1}{2!} + \dots + \frac{1}{k!}$. Show $\{S_k\}$ is Cauchy.

4. Let $0 < c < 1$. Suppose the sequence $\{a_n\}_{n=1}^{\infty}$ satisfies $|a_n - a_{n-1}| \le c\,|a_{n-1} - a_{n-2}|$ for all $n \ge 3$. Prove $\{a_n\}$ is Cauchy.

5. p. 57 #40

6. Consider the polynomial $p(x) = x^3 + 5x - 1$. It can be shown p has a unique solution in $(0,1)$. Take $a_1 \in (0,1)$ and define $a_{n+1} = \frac{1}{5}(1 - a_n^3)$.

(a) Prove $\{a_n\}$ is Cauchy.

(b) If $\lim_{n \to \infty} a_n = a$, prove $p(a) = 0$.

(c) Show that $a \in (0,1)$.

Bonus. (d) Prove that if $\exists\, 0 < C < 1$ s.t.

$$|a_n - a_{n-1}| \le C\,|a_{n-1} - a_{n-2}| \quad \forall n \ge 3,$$

then $|a - a_n| \le \frac{C^{n-1}}{1-C}\,|a_1 - a_2|$.

Bonus. (e) Take $a_1 = \frac{1}{2}$. Find n s.t. $|a - a_n| \le 10^{-4}$. Calculate a_n.