

Theorems whose proofs you should know for the Final Exam

1. If $\{a_n\} \rightarrow A$ and $\{b_n\} \rightarrow B$ then $\{a_n b_n\} \rightarrow A \cdot B$
2. A Cauchy sequence is bounded.
3. Every bounded sequence has a convergent subsequence.
4. If f is continuous at x_0 and g is continuous at $f(x_0)$, then $g \circ f$ is continuous at x_0 .
5. Bolzano's Thm - If $f: [a, b] \rightarrow \mathbb{R}$ is continuous, $f(a) < 0$ and $f(b) > 0$, then $\exists c \in (a, b)$ st. $f(c) = 0$.
6. E.V.T. If $f: [a, b] \rightarrow \mathbb{R}$ is cont. then $\exists c, d \in [a, b]$ st. $f(c) \leq f(x) \leq f(d) \forall x \in [a, b]$
7. Critical points theorem - Suppose $f: [a, b] \rightarrow \mathbb{R}$ has a relative (local) max or min at $c \in (a, b)$ and that f is diff at c . Then $f'(c) = 0$.
8. MVT (assuming Rolle's thm) If $f: [a, b] \rightarrow \mathbb{R}$ is cont and f is diff on (a, b) then $\exists c \in (a, b)$ st.
$$f'(c) = \frac{f(b) - f(a)}{b - a}$$
9. If $f: [a, b] \rightarrow \mathbb{R}$ is cont, then f is integrable.