

M 331 Assignment #4 (not to be handed in)

1. Use the def'n of convergence to prove

$$(a) \lim_{n \rightarrow \infty} \frac{2n+1}{4n+1} = \frac{1}{2}$$

$$(b) \lim_{n \rightarrow \infty} \frac{(-1)^n (n+1)}{n^2+1} = 0.$$

2. Show that  $\left\{ \frac{(-1)^n n}{n+1} \right\}$  diverges.

3. Decide if the following sequences are Cauchy

$$(a) \left\{ \frac{n+1}{n} \right\}$$

$$(b) \left\{ \frac{1 + (-1)^n n^2}{2n^2 + 3} \right\}$$

(Hint: Think about convergence.)

4. Let  $c_1 \in (0, 1)$  and set  $c_{n+1} = \frac{1}{5} (c_n^2 + 2)$ .

(a) Show that  $\{c_n\}$  is Cauchy.

(b) Find  $\lim_{n \rightarrow \infty} c_n$ .

5. Prove the following sequences are monotone and bounded. Find their limits.

$$(a) a_{n+1} = \sqrt{3a_n - 2}, \quad a_1 = \frac{3}{2}$$

$$(b) b_{n+1} = 2 - \frac{B}{b_n}, \quad b_1 = 2$$

where  $B \in (0, 1)$  is fixed.