

1. Decide if the following functions are uniformly continuous

(a)  $f(x) = \frac{x}{2+x}$  on  $[0, \infty)$

(b)  $g(x) = \sin \frac{1}{x}$  on  $(0, 1]$

2. Using the def'n of the derivative find the derivative of

(a)  $f(x) = \frac{x}{x^2+1}$

(b)  $g(x) = \sqrt{x+2}$  for  $x > -2$

3. Decide if the following functions are differentiable at the point  $x_0$ .

(a)  $f(x) = x|x|$  at  $x_0 = 0$ .

(b)  $g(x) = \begin{cases} \sin x & , x \in \mathbb{Q} \\ x & , x \notin \mathbb{Q} \end{cases}$  at  $x_0 = 0$ .

4. Let  $f(x) = \begin{cases} x^2 + 2 & \text{if } x \leq 2 \\ ax + b & \text{if } x > 2 \end{cases}$

(a) For what values of  $a, b$  is  $f$  continuous at  $x_0 = 2$ ?

(b) For what values of  $a, b$  is  $f$  differentiable at  $x_0 = 2$ ?

5. Let  $g(x) = \begin{cases} x^2 \sin \frac{1}{x} & , x \neq 0 \\ 0 & , x = 0 \end{cases}$

(a) Prove that  $g$  is differentiable at 0 and that  $g'(0) = 0$ .

Bonus (b) Show that  $g'$  is not continuous at 0